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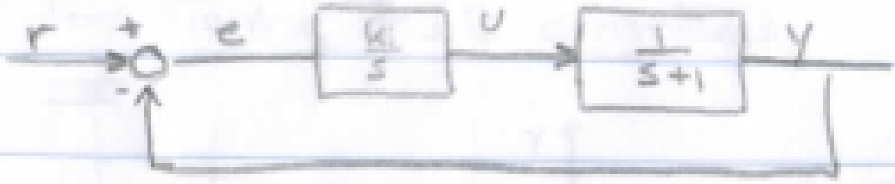
PID controllers – proportional-integral-derivative controllers

Integral Term



$$\frac{Y}{R} = \frac{\frac{k_i}{s}\left(\frac{1}{s+1}\right)}{1+\frac{k_i}{s}\left(\frac{1}{s+1}\right)} = \frac{k_i}{s^2+s+k_i}$$

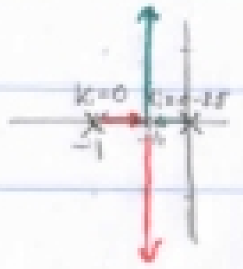
$r(t)=1 \rightarrow R(s)=\frac{1}{s}$

$$y_{ss} = \lim_{s\to 0} s\,Y(s) = s\,\frac{1}{s}\left(\frac{k_i}{s^2+s+k_i}\right) \rightarrow y_{ss} = \frac{k_i}{k_i} = 1$$

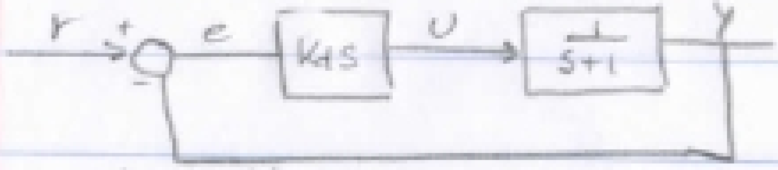
– integrator makes ss go to input value

$$s^2+s+k_i=0$$

$$s_{1,2} = \frac{-1 \pm \sqrt{1^2-4k_i}}{2}$$



Derivative Term



– Derivative term brings a "zero" or adds damping to counter affect of integrator which in turn will reduce oscillation

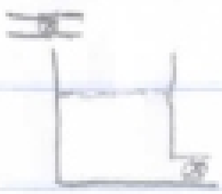
$$\frac{Y}{R} = \frac{(k_d s)\left(\frac{1}{s+1}\right)}{1+(k_d s)\left(\frac{1}{s+1}\right)} = \frac{k_d s}{s+1+k_d s} = \frac{k_d s}{(1+k_d)s+1}$$

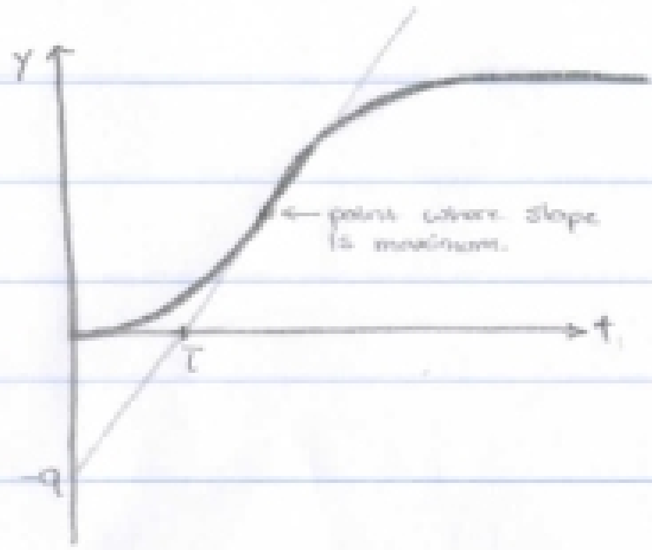
$$(1+k_d)s+1=0 \qquad s = \frac{-1}{1+k_d}$$

$r(t)=1 \rightarrow R(s)=\frac{1}{s}$

$$y_{ss} = s\,\frac{1}{s}\left(\frac{k_d s}{(1+k_d)s+1}\right) = 0$$

For Tuning PID controllers





$$\frac{U(s)}{E(s)} = k_p + \frac{k_i}{s} + k_d s = k_p\left(1 + \frac{k_i}{k_p s} + \frac{k_d}{k_p} s\right) = k_p\left(1 + \frac{1}{T_i}\frac{1}{s} + T_d s\right)$$