Compton Scattering

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In this paper, we describe our observations of Compton scattering. We used a 661.6 KeV γ-ray source to detect scattering of photons by electrons. We measured the energies of the scattered photons as well as the energies of the recoil electrons as a function of angle. The observed distribution matched, within reasonable error, to the theoretical angular distribution given by the Compton relation. Also, we looked at the total cross-section of Compton scattering by observing the attenuation of a beam of photons through plastic scintillator blocks. We found a total cross section of $(2.26 \pm 0.16) \times 10^{-25}$ cm^2/electron, compared to the 2.53×10^{-25} cm^2/electron from the Klein-Nishina value of the Compton cross section.

1. INTRODUCTION

One of the earliest successes of quantum mechanics were the successful explanations of the blackbody spectrum by Planck and the photoelectric effect by Einstein. Both of these works firmly established the idea that electromagnetic waves can be regarded as distinct quanta, each having energy $h\nu$ where h is the Planck constant and ν is the frequency of the wave. The experiment that gave the final confirmation of the validity of this hypothesis was the Compton scattering experiment.

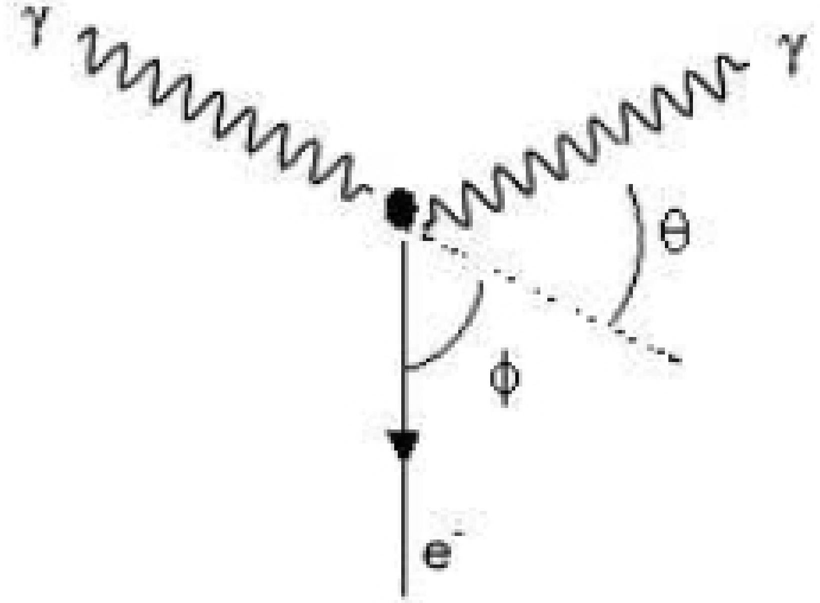

FIG. 1: Elastic scattering of photon by a free electron

The Compton effect was discovered by Arthur H. Compton in 1923. In his experiment, Compton irradiated a carbon target with an intense collimated beam of monochromatic molybdenum K_α X-rays and observed the wavelengths of the scattered X-rays as a function of angle. The wavelengths were measured precisely by using high-resolution X-ray spectrometry based on the Bragg angle for reflection of X-rays. Compton found that the radiation scattered through a given angle consisting of two components: one whose wavelength is the same as that of the primary incident radiation, while the other shifted relative to the incident wavelength by an amount that depends on the angle. This is inconsistent with classical theory, where only the intensity varies as function of $(1 + \cos^2\theta)$ and the wavelength is not shifted. Compton was able to explain the dependence on wavelength by treating the incoming radiation as a beam of particles of energy $h\nu$ with individual photons scattering elastically of individual electrons.

2. THEORETICAL DISCUSSION

2.1. The Compton Wavelength Shift

The Compton wavelength shift can be derived very simply from relativistic kinematics. Con-

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sider the situation shown in Figure 1. In an elastic collision, momentum as well as energy is conserved, and using this fact, one can derive that

$$\frac{1}{E'} = \frac{1}{E} + \frac{h}{m_e c^2}(1 - \cos\theta) \qquad (1)$$

where E and E' are the energies of the photon before and after the collision respectively. Treating the quanta of light as particles, we have thus arrived at a result which shows that light loses energy upon being scattered elastically by an electron. Photons also undergo Compton scattering by nuclei, but the shift in wavelength is very small since an atom is many thousands times more massive than an electron. The quantity $h/m_e c = 2.4 \times 10^-10 cm$ is called the Compton length; it is the wavelength shift when the scattering angle is 90^o.

2.2. The Scattering Cross Section

Classically the differential cross section for the scattering of radiation from electrons is given by:

$$\frac{d\sigma}{d\Omega} = r_0^2 \left(\frac{1 + \cos^2\theta}{2}\right) \qquad (2)$$

Here, r_0 is the classical electron radius, 2.82×10^{-13} cm. This relation can be easily derived by considering a linearly polarized plane wave incident on a charged particle. When integrated over all solid angles, we get the *Thomson* cross section:

$$\sigma_T = \frac{8\pi}{3} r_0^2 \qquad (3)$$

The classical derivation takes into account neither the recoil of the electron nor the relativistic effects nor the quantum effects. A full quantum field theoretic derivation (from [1]) yields the *Klein-Nishina* formula:

$$\frac{d\sigma}{d\Omega} = \frac{r_0^2}{2} \frac{1 + \cos^2\theta}{[1 + \gamma(1 - \cos\theta)]^2} \times \left[1 + \frac{\gamma^2(1 - \cos\theta)^2}{(1 + \cos^2\theta)[1 + \gamma(1 - \cos\theta)]}\right] \qquad (4)$$

Here, $\gamma = h\nu/m_e c^2$. By integrating the differential cross section given above over all solid angles, we get the following result for the total cross section:

$$\sigma_{KN} = 2\pi r_0^2 [\frac{1+\gamma}{\gamma^2}\left(\frac{2(1+\gamma)}{1+2\gamma} - \frac{1}{\gamma}\ln(1+2\gamma)\right) + \frac{1}{2\gamma}\ln(1+2\gamma) - \frac{1+3\gamma}{(1+2\gamma)^2}]$$

The Klein-Nishina cross section takes into account the electron recoil factor, relativistic quantum mechanics, and the interaction of the spin and magnetic moment of the electron with electromagnetic radiation.

3. EXPERIMENTAL OBSERVATION OF THE COMPTON SHIFT

A diagram for our arrangement is shown in Figure 2. Our apparatus consists of a beam of 661.6 KeV γ-rays from a 35-mCi Cesium-137 source, a target which also serves as the detector for recoil electrons, and a detector of the scattered photons. The range of 662 KeV photons is about 100 m in air and 4.7 cm in aluminum (from [2]). So, there is no need to enclose the detector in vacuum or to use a very thin target. Both detectors have a scintillator consisting of a 2" by 2" thallium-activated sodium-iodide optically coupled to a photomultiplier. When a photon hits the sodium iodide crystal, it scatters an electron from an iodide ion (because it has a high atomic number) and as the energetic photoelectron engages in Coulomb interactions with other electrons and nuclei in the sodium iodide crystal, it loses some of its energy via scintillation light. The intensity of light produced is closely proportional to the amount of energy dissipated in the process. Therefore, observation of the pulse-height spectrum from the photomultiplier enables a good measurement of the energy of the energy deposited.

The outputs from the two photomultipliers, after being amplified and appropriately inverted, are checked to see if they are in coinci-

FIG. 2: Experimental Setup for observing angular variation of energy

dence (that is, if they are with 500 ns of each other). If so, the multichannel analyzer accepts the scintillator pulse, otherwise it is not counted. This coincidence technique goes a long way in removing background signals and other irrelevant signals; without such electronic tricks, the pulses produced by Compton scattering would be buried in the background.

The calibration measurements was done using a sodium-22 source placed halfway between the target detector and scatter detector. Na^{22} generates a pair of 511-keV photons traveling in the opposite direction. We used this fact to ensure that the coincidence logic was working correctly. Our next step was to open the howitzer for the 662 KeV γ-rays. Then, we moved the scatter detector to various angles to determine the angular variation of the photon energy. A sample pulse-height spectrum for a scattering angle of 90^o is shown in Figure 3. As is shown for $\theta = 90^o$, we want to take the median energy for the peak for a range of angles and thus find the angular distribution of the photon energies. For us, this plan went well for the larger values of the scattering angle (with, correspondingly, low photon energies). But for $\theta < 45^o$, we could decipher no clear peak in our data. For example, in Fig-

FIG. 3: Pulse-height spectrum for $\theta = 90^o$