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Theory of Computability

The Class PSPACE

Section 8.2

PSPACE defined

Definition 8.6 **PSPACE** is the class of languages that are decidable in polynomial space on a deterministic TM. In other words,

$$\mathbf{PSPACE} = \mathbf{SPACE}(n) \cup \mathbf{SPACE}(n^2) \cup \mathbf{SPACE}(n^3) \cup \dots$$

NPSPACE can be defined similarly. However, the latter is not a very interesting class because, as an immediate corollary of Savitch's theorem, it coincides with **PSPACE** (squaring polynomial space again yields polynomial space).

This is what we know (why?):

$$\mathbf{P} \subseteq \mathbf{NP} \subseteq \mathbf{PSPACE} = \mathbf{NPSPACE} \subseteq \mathbf{EXPTIME}.$$

We, however, do not know whether any of the three \subseteq s can be replaced by $=$. Another set of huge open problems! It can be proven however that

$$\mathbf{P} \neq \mathbf{EXPTIME}.$$

So, at least one of the three containments must be proper (\subseteq but not $=$), even though we do not know which one(s)!