Introduction to Computer Engineering – EECS 203
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Introduction to number systems

Consider a base-10 number: $1,293$

$$1,293 = 1 \cdot 10^3 + 2 \cdot 10^2 + 9 \cdot 10^1 + 3 \cdot 10^0$$

For base-10, given an n-digit number in which d_i is the ith digit, the number is

$$\sum_{i=0}^{n} 10^{i-1} \cdot d_i$$

Introduction to number systems

This works for any base. Convert $2,012_3$ from base-3 to base-10.

$$2 \cdot 3^3 + 0 \cdot 3^2 + 1 \cdot 3^1 + 2 \cdot 3^0$$
$$2 \cdot 27 + 0 \cdot 9 + 1 \cdot 3 + 2 \cdot 1$$
$$54 + 0 + 3 + 2$$
$$59_{10}$$

Introduction to number systems

Convert 59_{10} from base-10 to base-3. Repeatedly divide by the greatest power of b (the base) that is less than the number.

Remainder	Try dividing	Digit	Comment
59	$3^4 = 81$	0	Too big
$59 - 0 \cdot 81 = 59$	$3^3 = 27$	2	O.K.
$59 - 2 \cdot 27 = 5$	$3^2 = 9$	0	Too big
$5 - 0 \cdot 9 = 5$	$3^1 = 3$	1	O.K.
$5 - 1 \cdot 3 = 2$	3^0	2	O.K.

$$02012_3 = 2012_3$$

Conversion works for any base

Review: For base-10, given an n-digit number in which d_i is the ith digit, the number is

$$\sum_{i=1}^{n} 10^{i-1} \cdot d_i$$

For base-b, given an n-digit number in which d_i is the ith digit, the number is

$$\sum_{i=1}^{n} \cdot b^{i-1} \cdot d_i$$

Useful bases

- 2: Also called *binary*. Most fundamental base in digital logic. Know this like the back of your hand.
- 8: Also called *octal*. Sometimes used by programmers. Prefer base 16.
- 10: Also called *decimal* or *Arabic*.
- 16: Also called *hexadecimal* or simple *hex*. One of the most compact and beautiful representations for digital computer programmers.

Binary

1	2	4	8	16	32	64	128	256	512	1,024 (1K)
2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8	2^9	2^{10}

$$\mathrm{k} \neq \mathrm{K}$$

$$1\,\mathrm{k} = 10^3 = 1{,}000$$
$$1\,\mathrm{K} = 2^{10} = 1{,}024$$

Decimal

- Most commonly used by human beings.
- Also called *Arabic*.
  - Actually developed in India and brought to Europe via Arabian empire.
- Largely replaced *Roman numerals*, which were more cumbersome when writing the large and complicated numbers used in astronomy and wide-spread trade.

Number systems

- Representation of positive numbers same in most systems
- A few special-purpose alternatives exist, e.g., Gray code
- Alternatives exist for signed numbers

Base-16: Hex

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F

Often prefixed with 0x.
What is 0xFF?

Reading assignment

- M. Morris Mano and Charles R. Kime. *Logic and Computer Design Fundamentals*. Prentice-Hall, NJ, third edition, 2004
- Sections 5.1–5.6

Computer geek culture reference

- Spelling things in ASCII (hex or binary)
- This is one of the lower forms of geek culture, akin to bad puns
- However, at least one university has things written into its buildings with subtle brick patterns in ASCII binary

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4a6934207375616e34206a6931207368653420
6a69342068656e332068616f332077616e3221
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