22S:30/105 Statistical Methods and Computing

More Nonparametric Methods

Lecture 25
April 27, 2011

Kate Cowles
374 SH, 335-0727
kcowles@stat.uiowa.edu

The Wilcoxon Signed-Rank Test

- for single sample or paired samples
- useful when the population distribution is not normal and the sample size is not large
  - of the within-pair differences in paired sample case or of individual values in single sample case
- makes use of the magnitudes of the differences as well as their signs

Example

Kashima, Baker, and Landen (1988) studied whether media-based instruction could help the parents of mentally handicapped children become more effective at teaching their children self-help skills.

As part of the study, 17 families participated in a training program. Before and after the training program, the primary parent took the Behavioral Vignettes test, which assesses knowledge of behavioral modification principles. A higher score indicates greater knowledge.

The following are the pre- and post-test training scores for 12 of their families:

Pre	Post
7	11
6	14
10	16
16	17
8	9
13	15
8	9
14	17
16	20
11	12
12	14
13	15

May we conclude from these data that the training program increases knowledge of behavior modification principles? (We will test at the $\alpha = .01$ level.)

Hypotheses of the Wilcoxon Signed Rank Test

The null hypothesis is that, in the underlying population of differences among pairs, the median difference is equal to 0.

$$H_0 : M_d = 0$$

The alternative hypothesis may be one- or two-sided.

$$H_a : M_d > 0$$
$$H_a : M_d < 0$$
$$H_a : M_d \neq 0$$

If we define our differences as post - pre, then our alternative would be:

$$H_a : M_d > 0$$

Steps in the Wilcoxon signed-rank procedure

1. Select a random sample of n pairs of observations.
2. Compute the difference d_i in each pair of observations. Delete all pairs in which $d_i = 0$, and reduce n accordingly.
3. Ignoring the signs of the d_is, rank their absolute values from smallest to largest. When there are ties in absolute values, assign each tied value the mean of the rank positions the tied values occupy.
4. Assign to each rank the sign of the d_i that yields that rank.

5. Find T_+, the sum of the ranks with positive signs, and T_-, the sum of the ranks with negative signs.
6. Let the test statistic T equal the smaller of T_+ and T_-.

Pre- and Post-Test example

Pre	Post	d_i	Rank
7	11	4	9.5
6	14	8	12
10	16	6	11
17	16	-1	-2.5
8	9	1	2.5
13	15	2	6
8	9	1	2.5
14	17	3	8
16	20	4	9.5
11	12	1	2.5
12	14	2	6
13	15	2	6

The sum of the negative ranks is $T = 2.5$.

SAS for the Wilcoxon Signed Rank Test

- carried out automatically by `proc univariate`
- SAS computes a slightly different form of the test statistic

$$S = \Sigma(positive\ ranks) - \frac{n(n+1)}{4}$$

recalling that n is the number of differences whose value is not equal to 0.

- computes p-value in two different ways depending on sample size
  - if $n \leq 20$, p-value is computed from each distribution of S, which can be enumerated under null hypothesis that distribution is symmetric around 0
  - when $n > 20$ approximate S is compared to approximate t distribution


```
data whatever ;
input pre post ;
diff = post - pre ;
datalines ;
 7  11
 6  14
10  16
16  17
 8   9
13  15
 8   9
14  17
16  20
11  12
12  14
13  15
;
run ;
```



```
proc univariate ;
var diff ;
run ;
```

```
                The UNIVARIATE Procedure
                     Variable:  diff

              Tests for Location: Mu0=0

Test              -Statistic-     -----p Value------

Student's t     t  4.521908     Pr > |t|     0.0009
Sign            M          6     Pr >= |M|    0.0005
Signed Rank     S         39     Pr >= |S|    0.0005
```

Note: For a 1-sided p-value, we would divide the 2-sided p-value by 2.

Interpreting the results

- Recall that we wanted to determine whether the audiovisual instruction improved parent's test scores.
- The null and alternative hypotheses regarding the median difference (that is, the median of post - pre), are

$$H_0 : M_d = 0$$
$$H_a : M_d > 0$$

- Can we reject H_0 at the .01 significance level?
- What does this mean with respect to the research question?