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Theory of Computability

Probabilistic algorithms

Section 10.2

Definition of probabilistic Turing machines

Definition 10.3 A *probabilistic Turing machine* M is a type of nondeterministic TM in which each nondeterministic step is called a *coin-flip step* and has two legal next moves. We assign a probability to each branch b of M 's computation on input w as follows. Define the probability of b to be

$$\Pr[b] = 2^{-k},$$

where k is the number of coin-flip steps that occur on branch b .

We define the probability that M accepts w to be

$$\Pr[M \text{ accepts } w] = \sum_{\substack{b \text{ is an} \\ \text{accepting} \\ \text{branch}}} \Pr[b]$$

In other words, the probability that M accepts w is the probability that we would reach an accepting configuration if we simulated M on w by flipping a coin to determine which move to follow at each coin-flip step. We let

$$\Pr[M \text{ rejects } w] = 1 - \Pr[M \text{ accepts } w]$$

