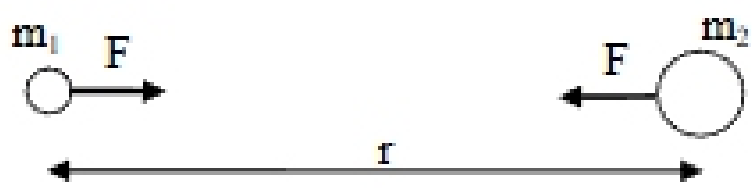
Gravity

Newton's Universal Law of Gravitation (first stated by Newton): any two masses m_1 and m_2 exert an attractive gravitational force on each other according to

$$F = G\frac{m_1 m_2}{r^2}$$



This applies to all masses, not just big ones.

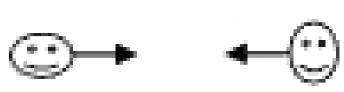
G = universal constant of gravitation = 6.67×10^{-11} N m^2 / kg^2 (G is very small, so it is very difficult to measure!)

Don't confuse G with g: "Big G" and "little g" are totally different things.

Newton showed that the force of gravity must act according to this rule in order to produce the observed motions of the planets around the sun, of the moon around the earth, and of projectiles near the earth. He then had the great insight to realize that this same force acts between *all* masses. [That gravity acts between all masses, even small ones, was experimentally verified in 1798 by Cavendish.]

Newton couldn't say *why* gravity acted this way, only *how*. Einstein (1915) General Theory of Relativity, explained why gravity acted like this.

Example: Force of attraction between two humans. 2 people with masses $m_1 \cong m_2 \cong 70$ kg, distance r = 1 m apart.

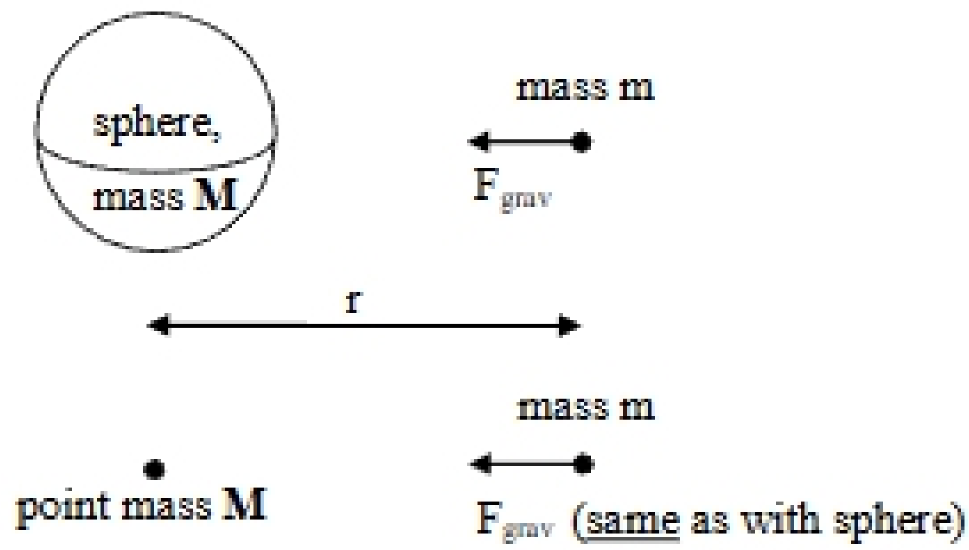


$$F = G\frac{m_1 m_2}{r^2} = \frac{(6.67\times10^{-11})(70)^2}{1^2} = 3.3\times10^{-7}\ \text{N}$$

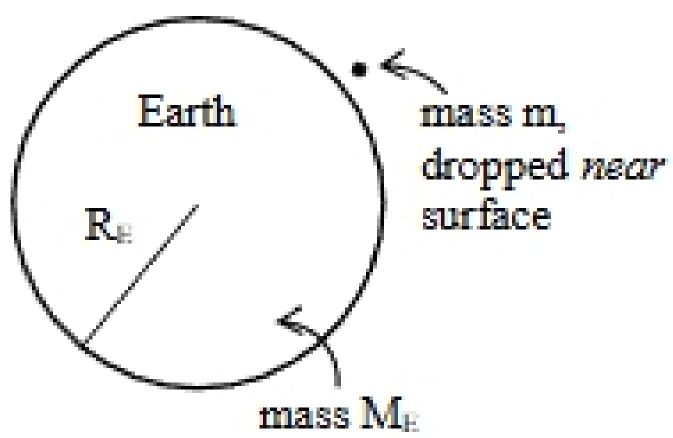
This is a very tiny force! It is the weight of a mass of 3.4×10^{-5} gram. A hair weighs 2×10^{-3} grams – the force of gravity between two people talking is about 1/60 the weight of a single hair.

Computation of g

Important fact about the gravitational force from spherical masses: a spherical body exerts a gravitational force on surrounding bodies that is the same as if all the sphere's mass were concentrated at its center. This is difficult to prove (Newton worried about this for 20 years.)



We can now *compute* the acceleration of gravity g ! (Before, g was experimentally determined, and it was a mystery why g was the same for all masses.)



$$F_{grav} = m\,a = m\,g$$

$$G\frac{M_E m}{R_E^{\,2}} = m\,g$$

(since $r = R_E$ is distance from m to center of Earth)

m's cancel ! $\Rightarrow$ $\boxed{g = \dfrac{G M_E}{R_E^{\,2}}}$

If you plug in the numbers for G, M_E, and R_E, you get $g = 9.8\ \text{m/s}^2$.

Newton's Theory explains why all objects near the Earth's surface fall with the same acceleration (because the m's cancel in $F_{grav} = \frac{G M m}{R^2} = m\,a$.) Newton's theory also makes a quantitative prediction for the value of g, which is correct.

Example: g on Planet X. Planet X has the same mass as earth ($M_X = M_E$) but has ½ the radius ($R_X = 0.5\,R_E$). What is g_x , the acceleration of gravity on planet X?

Planet X is denser than earth, so expect g_x larger than g.

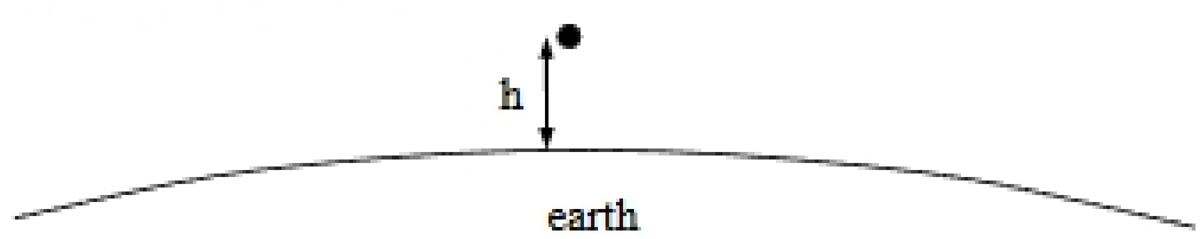
$$g_x = \frac{G M_X}{R_X^{\,2}} = \frac{G M_E}{\left(R_E/2\right)^2} = \frac{1}{1/2^{\,2}}\underbrace{\frac{G M_E}{R_E^{\,2}}}_{\text{g of earth}} = 4\,g\,.$$

Don't need values of G, M_E, and R_E!

Method II, set up a ratio:

$$\frac{g_X}{g_E} = \frac{\left(\frac{G M_X}{R_X^{\ 2}}\right)}{\left(\frac{G M_E}{R_E^{\ 2}}\right)} = \frac{M_X}{M_E}\left(\frac{R_E}{R_X}\right)^2 = 1 \cdot 2^2 = 4 \ , \quad g_X = 4 g_E$$

At height h above the surface of the earth, g is less, since we are further from the surface, further from the earth's center.



$$r = R_E + h \ \Rightarrow$$

$$g = \frac{G M_E}{r^2} = \frac{G M_E}{(R_E + h)^2}$$

The space shuttle orbits earth at an altitude of about 200 mi × 1.6 km/mi ≅ 320 km. Earth's radius is R_E = 6380 km. So the space shuttle is only about 5% further from the earth's center than we are. If r is 5% larger, then r^2 is about 10% larger, and

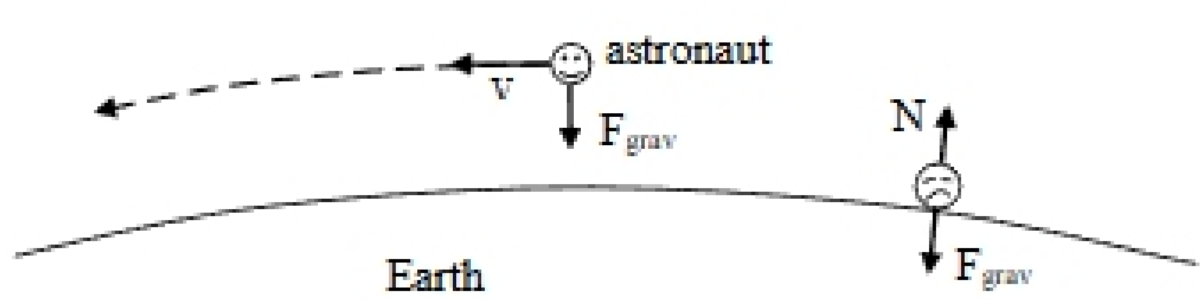
$$F_{grav}(\text{on mass m in shuttle}) = G\frac{M_E\, m}{(R_E + h)^2} \cong \text{about 10\% less than on earth's surface}$$

Astronauts on the shuttle experience almost the same F_{grav} as when on earth. So why do we say the astronauts are *weightless*??

"Weightless" does not mean "no weight".

"Weightless" means "freefall" means the only force acting is gravity.

If you fall down an airless elevator shaft, you will feel exactly like the astronauts. You will be weightless, you will be in free-fall.



An astronaut falls toward the earth, as she moves forward, just as a bullet fired horizontally from a gun falls toward earth.

Orbits

Consider a planet like Earth, but with no air. Fire projectiles horizontally from a mountain top, with faster and faster initial speeds.

