Demos: drift velocity
ohm's Law: $V = iR$

Ch. 29

Current & Resistance

Static situation

Electric field inside a conductor is zero even in an applied external field. If not, $F_E = qE_{ext} = ma \Rightarrow$ charges accelerate (not static situation)

$\downarrow\downarrow\downarrow \vec{E}_{ext}$

$- - - -$

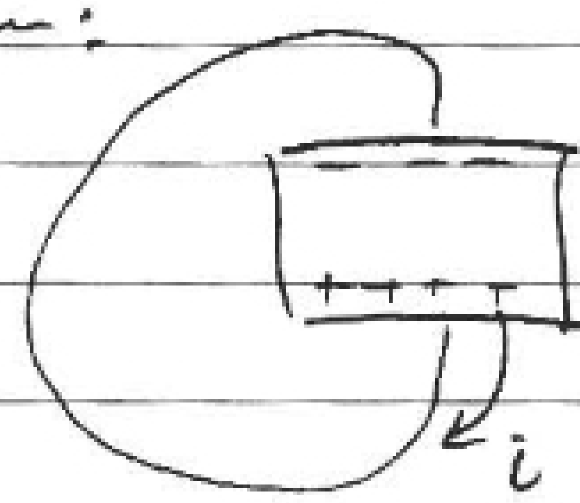
$\uparrow E_{int} \Rightarrow \vec{E} = \vec{E}_{ext} + \vec{E}_{int} = 0$

$+ + + + + +$

charges moved by electric field

Dynamic

Now suppose we have a closed loop so that charges can be drained from bottom:



$\Rightarrow$ electric current $i = \dfrac{dq}{dt}$ $\quad \dfrac{\Delta Q}{\Delta T}$

direction of (+) charge

$=$ amount of charge dq passing through an area in a time dt

unit $=$ 1 ampere $=$ 1 Coulomb/sec

The net charge passing through any surface is $q = \int i\,dt$

The current density $\vec{J}$ is the current per unit area passing through a surface: $j = i/A$

Take $\vec{J}$ direction as current direction

$\vec{J}$ points in direction of current

$i = \int \vec{j} \cdot d\vec{A}$ $\quad$ = current passing through surface

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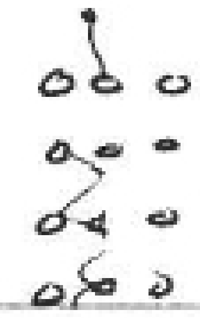


Demo: Drift velocity, showing steady-state drift

Current drift velocity

- metal is a lattice of fixed ⊕ ions & free "valence" electrons ⊖
- Electrons in motion due to thermal energy

> Paul Drude Model ~1900 ⇒ $\bar{v} = \sqrt{\frac{8kT}{\pi m_e}} \sim 10^5 \, m/s$ @ T=300K ($3\times10^{-4}c$)

Apply electric field ⇒ electrons accelerate

$$\vec{a} = \frac{(-e)\vec{E}_{ext}}{m_e}$$

$F_e = eE = ma$

but electrons eventually scatter off ions

⇒ Net effect is a small constant drift velocity in direction of field, with lots of scatters (like gentle breeze of air molecules)

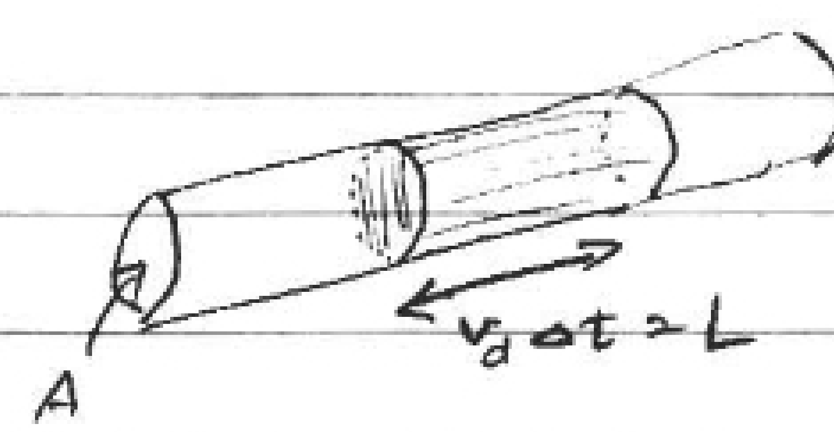
DEMO

current: $i = \frac{\Delta Q}{\Delta t} = \frac{e \cdot n \cdot V}{\Delta t}$

n = # density of valence electrons

V = volume

$$i = \frac{en(A v_d \Delta t)}{\Delta t}$$



$$i = env_d A$$

current density

$$\boxed{J = \frac{i}{A} = env_d}$$

More generally $i = \int \vec{J} \cdot d\vec{A}$

$i = JA$ for uniform current density

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Ex: Estimate V_d for copper:

Copper has $\approx 1 e^-/\text{atom}$ free

$n = \rho \frac{N_A}{M}$ $\quad \rho = 8.92\ g/cm^3 \quad M = 63.5 g/mol$

$n = 8.5 \times 10^{22}/cm^3 = \underline{8.5 \times 10^{28}\ m^{-3}}$

Suppose $i = 1A$ in a wire with $r = 0.815 mm$

$V_d = \frac{I}{Ane} = \frac{1\ C/s}{\pi (8.15 \times 10^{-4} m)^2 (8.5 \times 10^{28} m^{-3})(1.6 \times 10^{-19} C)}$

$\underline{V_d = 3.5 \times 10^{-5}\ m/s}$ $= 0.035\ mm/s$! $\quad$ 10^{10} times smaller than thermal motion

$= 2 mm/min$

Resistivity

V_d is proportional to $\vec{E}$

$\Rightarrow \vec{j} = env_d = \sigma \vec{E}$ $\quad \boxed{\vec{J} = \sigma \vec{E}}$ ohm's Law

$\sigma \equiv$ conductivity $=$ Siemen/m

1 siemans $=$ 1 amp/volt

$\underline{\rho \equiv \frac{1}{\sigma} = \text{resistivity}}$ $\quad \rho = \text{ohm-m}\ (\Omega\text{-m})$

$\Rightarrow \vec{E} = \rho \vec{J}$

ohm's Law $\Rightarrow$ resistivity is a property of the material, not of applied field

$\rho \neq f(j, E)$