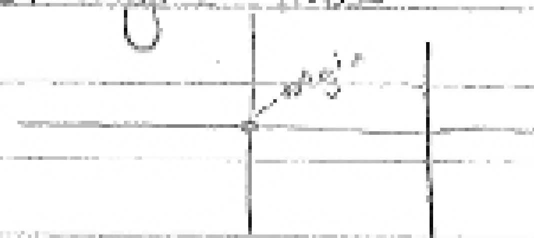
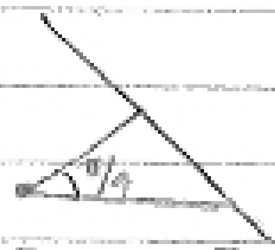
Straight lines:



$x=5 \Rightarrow r\cos\theta = 5 \qquad r = \frac{5}{\cos\theta} = \boxed{5\sec\theta}$

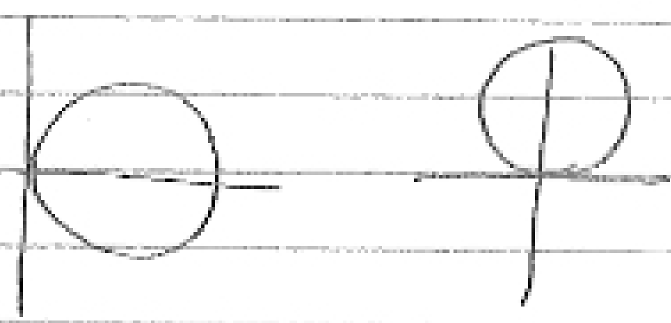


$r = 5\sec(\theta - \pi/4)$

$ax + by = c$

$ar\cos\theta + br\sin\theta = c$

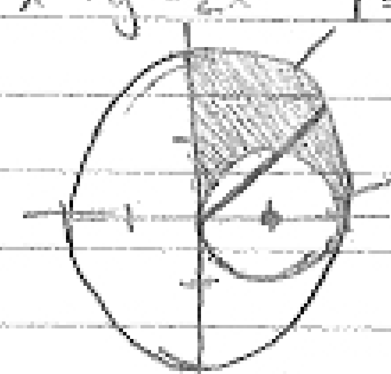
$r(a\cos\theta + b\sin\theta) = c$



Those are good in polar too!

10/23/14

$\iint_R x\,dA$ In the 1st quadrant between $x^2+y^2=4$ & $x^2+y^2=2x$



$x^2-2x+1+y^2=1 \qquad x^2+y^2=2x$

$(x-1)^2+y^2=1 \qquad r^2=2r\cos\theta$

$r=2\cos\theta$

$$\int_0^{\pi/2}\int_{2\cos\theta}^{2} r\cos\theta\, r\,dr\,d\theta = \int_0^{\pi/2}\int_{2\cos\theta}^{2} r^2\cos\theta\,dr\,d\theta$$

$$\int \tfrac{1}{3}\cos\theta\, r^3 \Big|_{2\cos\theta}^{2} d\theta$$

$$\int_0^{\pi/2} \frac{\cos\theta}{3}(8-8\cos^3\theta)\,d\theta$$

$$\tfrac{8}{3}\int \cos\theta - \cos^4\theta\,d\theta$$

$$\tfrac{8}{3}\int \cos\theta - (\cos^2\theta)^2\,d\theta$$

$$\tfrac{8}{3}\int_0^{\pi/2} \cos\theta - \left(\frac{1+\cos(2\theta)}{2}\right)^2 d\theta$$

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$$\frac{8}{3}\int_0^{\pi/2} \cos\theta - \frac{1}{4}\left(1 + 2\cos(2\theta) + \cos(2\theta)^2\right) d\theta$$

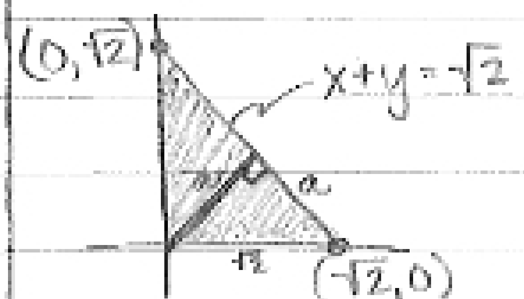
$$\frac{8}{3}\int_0^{\pi/2} \cos\theta - \frac{1}{4} - \frac{1}{2}\cos(2\theta) - \frac{1}{4}\left(\frac{1+\cos(4\theta)}{2}\right) d\theta$$

$$= \frac{8}{3}\left[\sin\theta - \frac{\theta}{4} - \frac{1}{4}\sin(2\theta) - \frac{1}{4}\left(\frac{\theta + \frac{\sin(4\theta)}{4}}{2}\right)\right]_0^{\pi/2}$$

$$= \frac{8}{3}\left(1 - \frac{\pi}{8} - 0 - \frac{1}{4}\left(\frac{\pi/2 + 0}{2}\right)\right)$$

$$= \frac{8}{3}\left(1 - \frac{3\pi}{16}\right) = \boxed{\frac{8}{3} - \frac{\pi}{2}}$$

$$\iint_A dA = ? = \sqrt{2}\cdot\sqrt{2}\cdot\frac{1}{2} = \boxed{1}$$



$a^2 + a^2 = 2$

$a = 1$

* Now lets do it the hard way (:

$r\cos\theta = 1 \quad r = \sec\theta$

$r = \sec(\theta \pm \pi/4)$

$r(\pi/4) = 1$

$\times\; r(\pi/4) = \sec(\pi/4 + \pi/4) = \frac{1}{\cos(\pi/2)} = \frac{1}{0}$

$r = \sec(\theta + \pi/4)$

$\checkmark\; r(\pi/4) = \sec(\pi/4 - \pi/4) = \frac{1}{\cos 0} = \frac{1}{1}$

$$\int_0^{\pi/2}\int_0^{\sec(\theta-\pi/4)} r\,dr\,d\theta$$

$$\int_0^{\pi/2} \frac{1}{2}r^2\Big|_0^{\sec(\theta-\pi/4)} d\theta = \frac{1}{2}\int_0^{\pi/2} \sec^2(\theta - \pi/4) - 0\, d\theta$$

$$= \frac{1}{2}\tan(\theta - \pi/4)\Big|_0^{\pi/2}$$

$$= \frac{1}{2}\left(\tan(\pi/2 - \pi/4) - \tan(0 - \pi/4)\right)$$

$$= \frac{1}{2}(1 - (-1)) = \boxed{1}$$

Area inside both curves (Hint: these are circles)

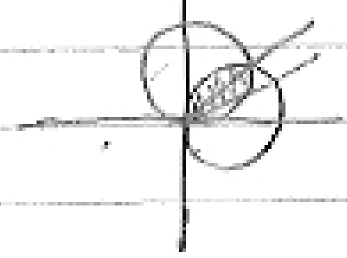
$r = \cos\theta$ & $r = \sin\theta$

$r^2 = r\cos\theta$

$x^2 + y^2 = x$

$x^2 - x + 1/4 + y^2 = 1/4$

$(x - 1/2)^2 + y^2 = 1/4$



$r^2 = r\sin\theta$

$x^2 + y^2 = y$

$x^2 + y^2 - y - 1/4 = 1/4$

$(y - 1/2)^2 + x^2 = 1/4$

$$\iint_A dA = 2\int_0^{\pi/4}\int_0^{\sin\theta} r\,dr\,d\theta$$

$$= \cancel{2}\int_0^{\pi/4} \tfrac{1}{2} r^2 \Big|_0^{\sin\theta} d\theta = \int_0^{\pi/4} \sin^2\theta \, d\theta$$

$$= \int_0^{\pi/4} \frac{1 - \cos(2\theta)}{2}\, d\theta$$

$$= \frac{\theta - \sin(2\theta)\frac{1}{2}}{2} \Big|_0^{\pi/4}$$

$$= \frac{\pi/4 - \sin(\pi/2)\frac{1}{2}}{2} - 0 = \frac{\pi/4 - 1/2}{2}$$

$$= \boxed{\frac{\pi}{8} - \frac{1}{4}} = \boxed{\frac{\pi - 2}{8}}$$

$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$ Just for your dining and dancing pleasure