EE468G NOTES (4)

Conductors:

There are free charges

Conduction current: $\vec{J} = \sigma \vec{E}$ (General Ohm's law)

σ : conductivity [S/m]

Insider perfect conductor: $\sigma = \infty$, $\vec{E} = 0$, $\vec{J} = 0$,

Power dissipation:

$$P = \int_V \sigma \left|\vec{E}\right|^2 dV$$

Dielectrics:

Charges are tightly bound to individual nuclei.

Then can only move a fraction of an atomic distance in a electric field.

Un-polarized molecule

Polarized molecule

For isotropic dielectric, the polarization vector is related to electric field by

$$\vec{P} = \varepsilon_0 \chi_e \vec{E}\,, \qquad \chi_e : \text{Susceptibility (no unit)}$$

Charges associated with the polarization vector:

$$\rho_{VP} = -\nabla \cdot \vec{P}, \qquad [\mathrm{C/m^3}]$$

$$\rho_{SP} = \hat{n} \cdot \vec{P}, \qquad [\mathrm{C/m^2}]$$

Maxwell's equation in dielectric material

$$\nabla \cdot \varepsilon_0 \vec{E} = \rho_V + \rho_{VP} = \rho_V - \nabla \cdot \vec{P}$$

$$\nabla \cdot \varepsilon_0 \vec{E} + \nabla \cdot \vec{P} = \rho_V$$

$$\nabla \cdot \left[\varepsilon_0 \vec{E} + \vec{P} \right] = \rho_V$$

Let $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}$, then we have

$\nabla \cdot \vec{D} = \rho_V$, ρ_V ---- Free-charge density.

Since $\vec{P} = \varepsilon_0 \chi_e \vec{E}$,

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 \vec{E} + \varepsilon_0 \chi_e \vec{E} = \varepsilon_0 (1 + \chi_e) \vec{E} = \varepsilon_0 \varepsilon_r \vec{E}$$

$\varepsilon = \varepsilon_0 \varepsilon_r$ is the dielectric permittivity

ε_r is the relative permittivity.

Air is very close to free-space, hence, for air, $\varepsilon_{\text{r, air}} \approx 1$