

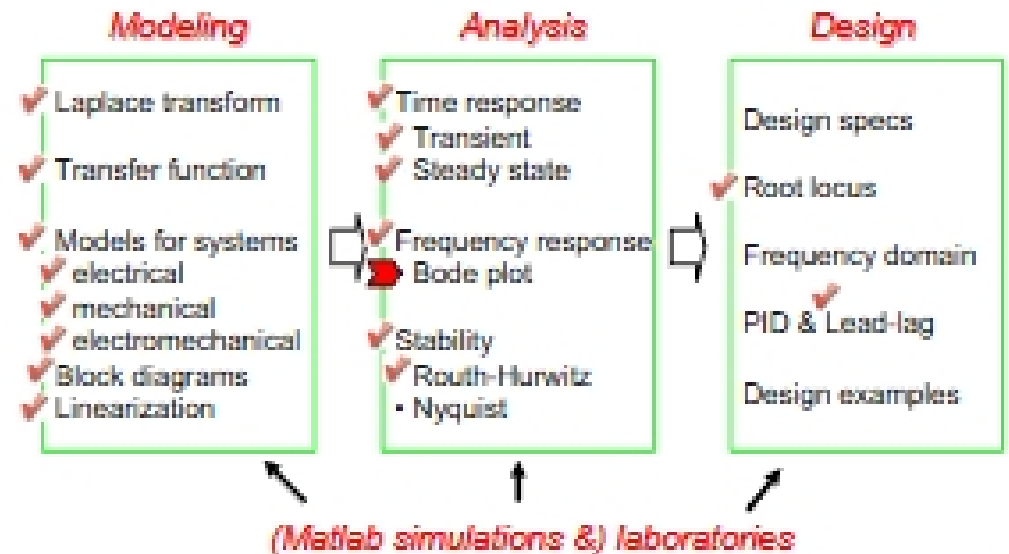
ME451: Automatic Control

Lecture 24

Bode diagram of connected systems

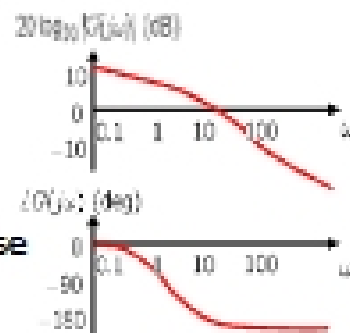
Dr. Jongeun Choi
 Department of Mechanical Engineering
 Michigan State University

Course roadmap



Sketching Bode plot

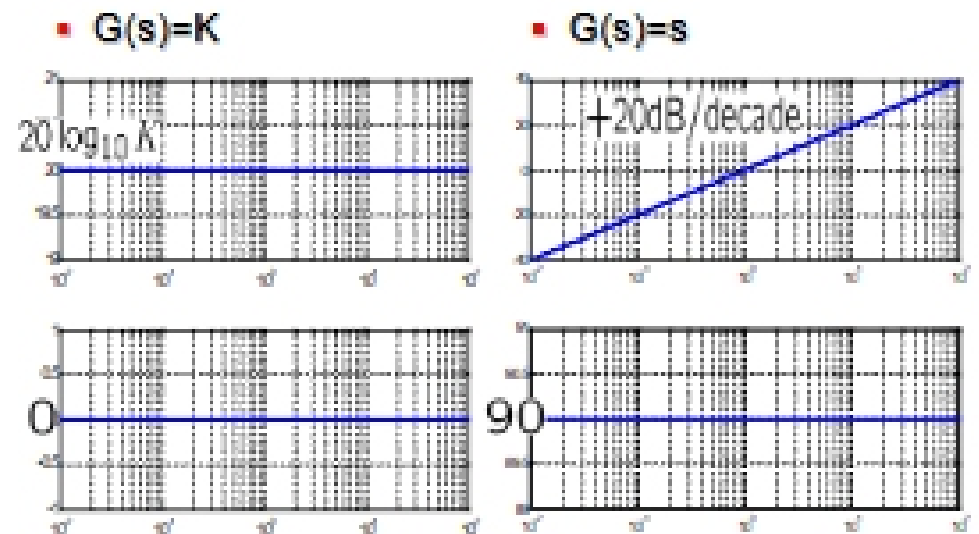
- Basic functions
 - ✓ Constant gain
 - ✓ Differentiator and integrator
 - ✓ Double integrator
 - ✓ First order system and its inverse
 - Second order system
 - Time delay



- Product of basic functions
 1. Sketch Bode plot of each factor, and
 2. Add the Bode plots graphically.

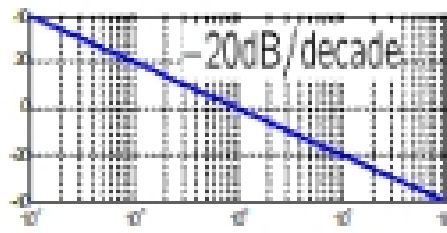
Main advantage of Bode plot!

Bode plot of basic functions (review)

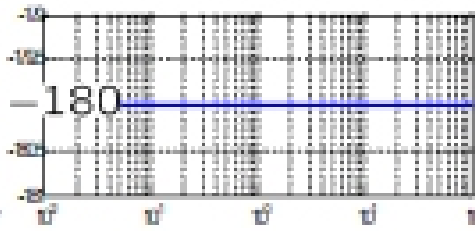
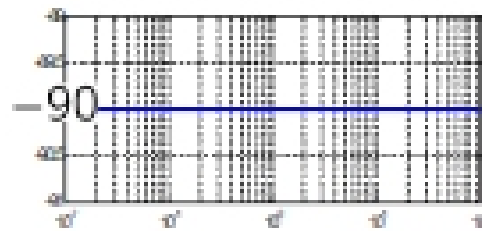
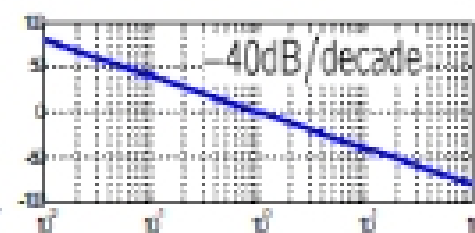


Bode plot of basic functions (review)

▪ $G(s)=1/s$



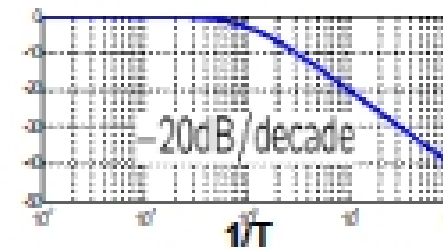
▪ $G(s)=1/(s^2s)$



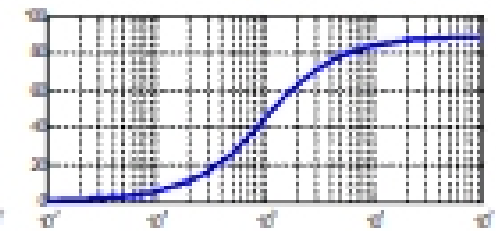
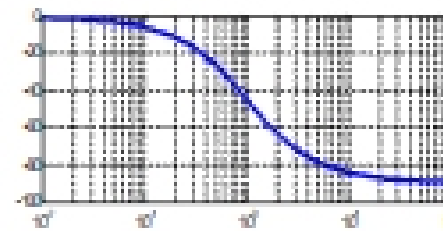
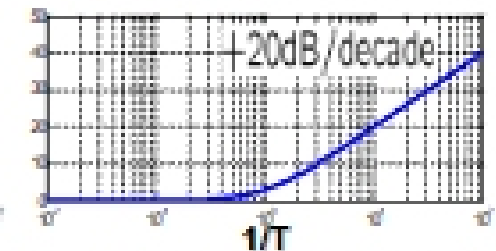
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Bode plot of basic functions (review)

▪ $G(s)=1/(Ts+1)$

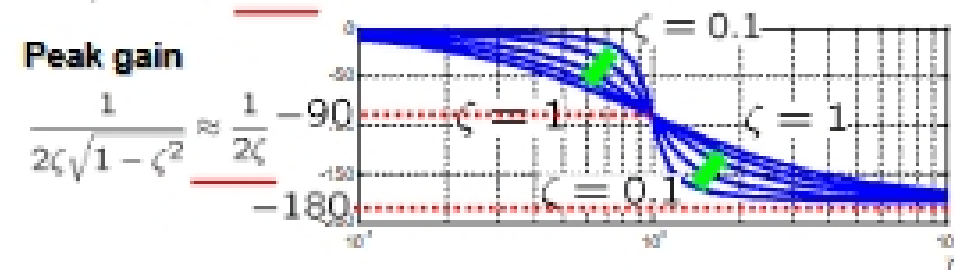
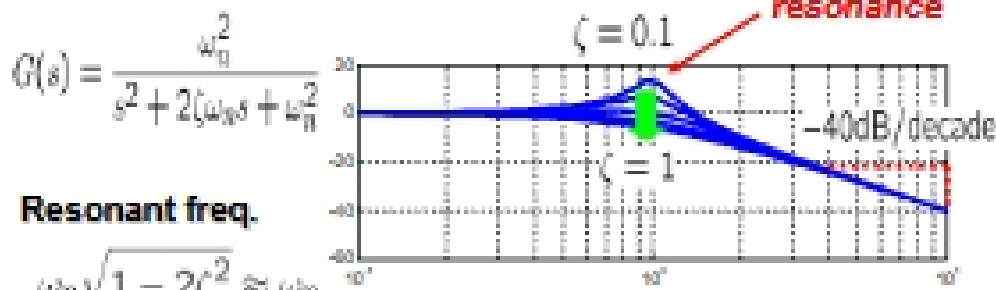


▪ $G(s)=Ts+1$



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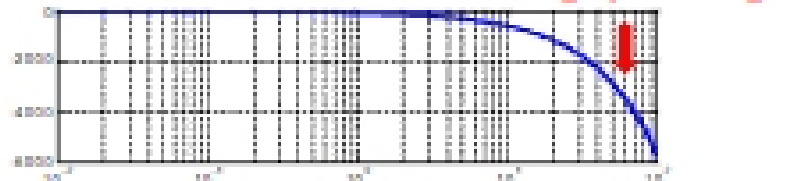
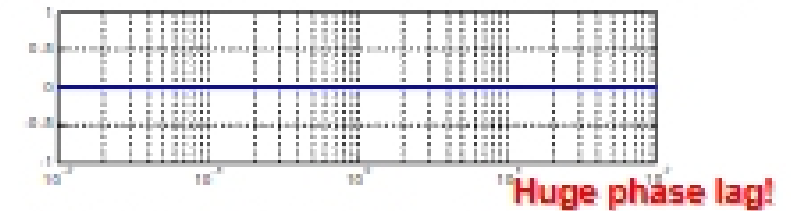
Bode plot of a 2nd order system



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Bode plot of a time delay

$G(s) = e^{-Ts} \Rightarrow |G(j\omega)| = 1, \forall \omega, \angle G(j\omega) = -\omega T(\text{rad})$



The phase lag causes instability of the closed-loop system, and thus, the difficulty in control.

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An advantage of Bode plot

- Bode plot of a series connection $G_1(s)G_2(s)$ is the addition of each Bode plot of G_1 and G_2 .

- Gain

$$20 \log_{10} |G_1(j\omega)G_2(j\omega)| = 20 \log_{10} |G_1(j\omega)| + 20 \log_{10} |G_2(j\omega)|$$

- Phase

$$\angle G_1(j\omega)G_2(j\omega) = \angle G_1(j\omega) + \angle G_2(j\omega)$$

- We use this property to design $C(s)$ so that $G(s)C(s)$ has a "desired" shape of Bode plot.

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Short proofs

- Use polar representation

$$G_1(j\omega) = |G_1(j\omega)| e^{j\angle G_1(j\omega)} \quad G_2(j\omega) = |G_2(j\omega)| e^{j\angle G_2(j\omega)}$$

$$\begin{aligned} \text{Then, } G_1(j\omega)G_2(j\omega) &= |G_1(j\omega)||G_2(j\omega)| e^{j\angle G_1(j\omega)} e^{j\angle G_2(j\omega)} \\ &= |G_1(j\omega)||G_2(j\omega)| e^{j\{\angle G_1(j\omega) + \angle G_2(j\omega)\}} \end{aligned}$$

Therefore,

$$20 \log_{10} |G_1(j\omega)G_2(j\omega)| = 20 \log_{10} |G_1(j\omega)| + 20 \log_{10} |G_2(j\omega)| = 20 \log_{10} |G_1(j\omega)| + 20 \log_{10} |G_2(j\omega)|$$

$$\angle G_1(j\omega)G_2(j\omega) = \angle G_1(j\omega) + \angle G_2(j\omega)$$

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Example 1

- Sketch the Bode plot of a transfer function

$$G(s) = \frac{10}{s}$$

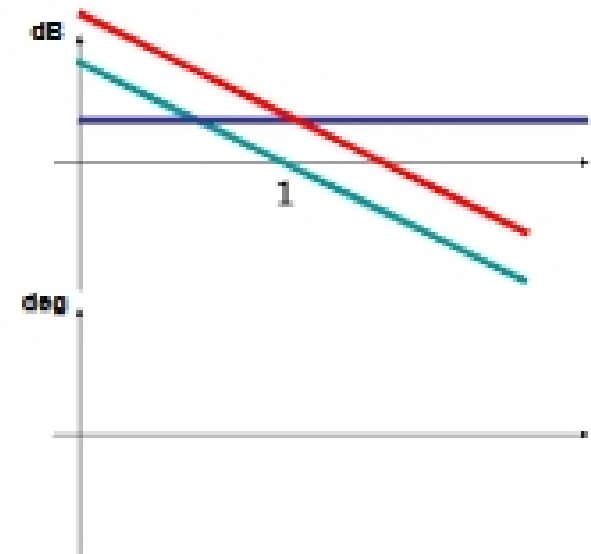
1. Decompose $G(s)$ into a product form:

$$G(s) = 10 \cdot \frac{1}{s}$$

2. Sketch a Bode plot for each component on the same graph.
3. Add them all on both gain and phase plots.

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Example 1 (cont'd)



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