Constraint Satisfaction

Reading: Russell & Norvig Chapter 5,
Kumar, "Algorithms for constraint satisfaction problems: A survey"

Overview

- Constraint Processing offers a powerful problem-solving paradigm
  - View a problem as a **set of variables** to which we have to assign **values** that satisfy a number of **problem-specific constraints**.
  - Constraint programming, CSPs, constraint logic programming…
- Algorithms for CSPs
  - Backtracking (systematic search)
  - Constraint propagation (k-consistency)
  - Variable ordering heuristics
  - Backjumping and dependency-directed backtracking

Informal Definition of CSP

- CSP = Constraint Satisfaction Problem
- Given
  (1) a finite set of variables
  (2) each with a domain of possible values (often finite)
  (3) a set of constraints that limit the values the variables can take on
- A **solution** is an assignment of a value to each variable such that the constraints are all satisfied.
- Tasks might be to decide if a solution exists, to find a solution, to find all solutions, or to find the "best solution" according to some metric.

SEND + MORE = MONEY

Assign distinct digits to the letters

S, E, N, D, M, O, R, Y

such that

```
  S E N D
+ M O R E
---------
= M O N E Y
```

holds.

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holds.

Solution

```
  9 5 6 7
+ 1 0 8 5
---------
= 1 0 6 5 2
```

Modeling

Formalize the problem as a *constraint problem*:

- number of variables: n
- constraints: $c_1, \ldots, c_m \subseteq Z^n$
- problem: Find $a = (v_1, \ldots, v_n) \in Z^n$ such that $a \in c_i$, for all $1 \leq i \leq m$

A Model for MONEY

- number of variables: 8
- constraints:

$c_1 = \{(S,E,N,D,M,O,R,Y) \in \mathbb{Z}^8 \mid 0 \le S,\ldots,Y \le 9\}$

$c_2 = \{(S,E,N,D,M,O,R,Y) \in \mathbb{Z}^8 \mid$

$1000*S + 100*E + 10*N + D$

$+ 1000*M + 100*O + 10*R + E$

$= 10000*M + 1000*O + 100*N + 10*E + Y\}$

A Model for MONEY (continued)

- more constraints

$c_3 = \{(S,E,N,D,M,O,R,Y) \in \mathbb{Z}^8 \mid S \neq 0\}$

$c_4 = \{(S,E,N,D,M,O,R,Y) \in \mathbb{Z}^8 \mid M \neq 0\}$

$c_5 = \{(S,E,N,D,M,O,R,Y) \in \mathbb{Z}^8 \mid S..Y$ all different$\}$

Solution for MONEY

$c_1 = \{(S,E,N,D,M,O,R,Y) \in \mathbb{Z}^8 \mid 0 \le S,\ldots,Y \le 9\}$

$c_2 = \{(S,E,N,D,M,O,R,Y) \in \mathbb{Z}^8 \mid$

$1000*S + 100*E + 10*N + D$

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$c_5 = \{(S,E,N,D,M,O,R,Y) \in \mathbb{Z}^8 \mid S..Y$ **all different**$\}$

Solution: $(9,5,6,7,1,0,8,2) \in \mathbb{Z}^8$