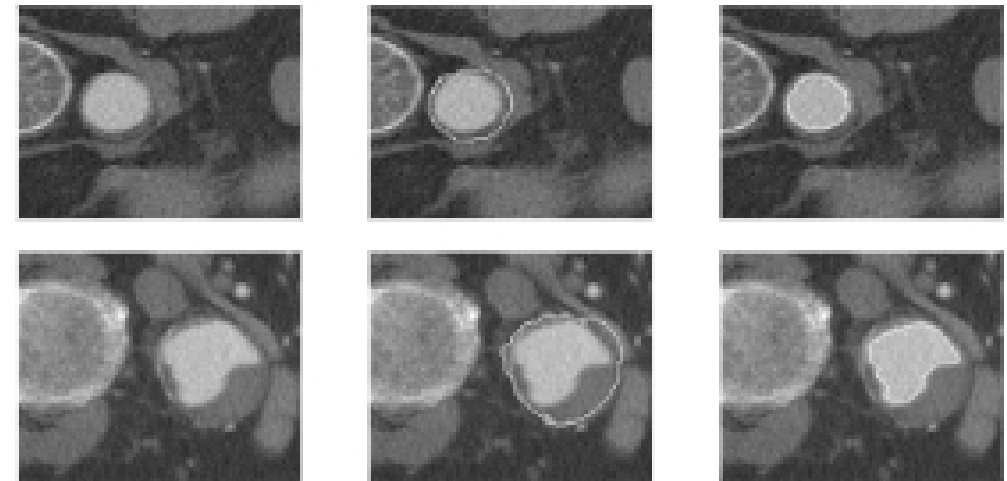


Introduction (1)

- The active contour model, or **snake**, is defined as an energy-minimizing spline.
- Active contours results from work of **Kass et.al.** in 1987.
- Active contour models may be used in image segmentation and understanding.
- The snake's energy depends on its shape and location within the image.
- Snakes can be closed or open

Active Snakes and their applications - Julien Tonier - Sem 201 Fall 2012

Introduction (2)



Aorta segmentation using active contours

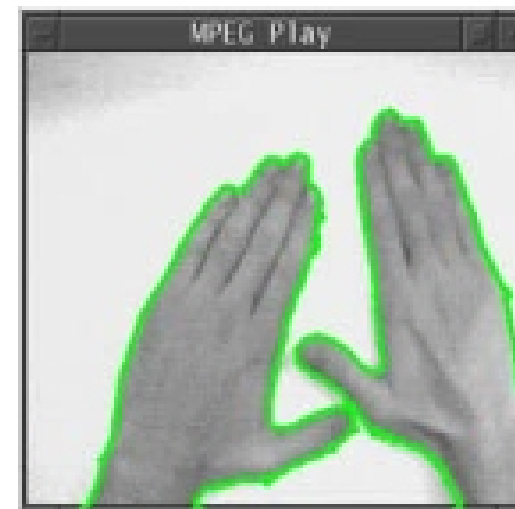
Active Snakes and their applications - Julien Tonier - Sem 201 Fall 2012

Introduction (3)

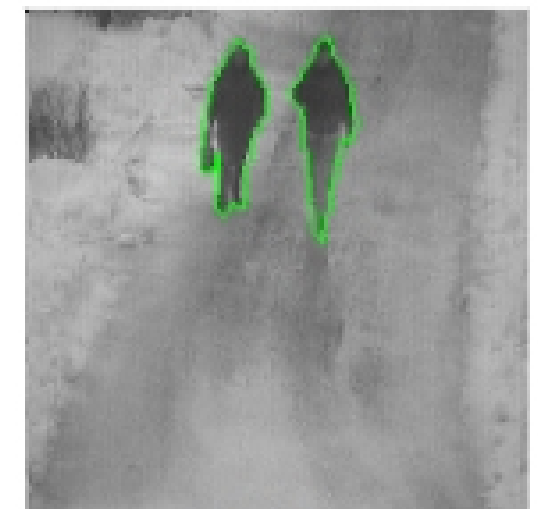
- First an initial spline (snake) is placed on the image, and then its energy is minimized.
- Local minima of this energy correspond to desired image properties.
- the snake is active, always minimizing its energy functional, therefore exhibiting dynamic behavior.
- Also suitable for analysis of dynamic data or 3D image data.

Active Snakes and their applications - Julien Tonier - Sem 201 Fall 2012

Examples (1)



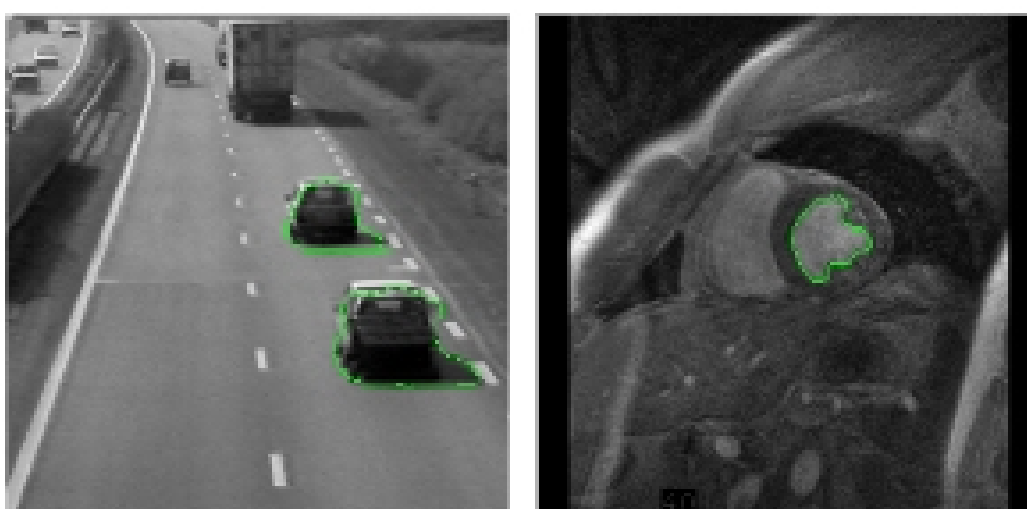
Hands



Peopl

Active Snakes and their applications - Julien Tonier - Sem 201 Fall 2012

Examples (2)



Highway

Heart

Active Snakes and their applications - Julien Tonier - Sem 201 Fall 2012

Modeling

- The contour is defined in the (x, y) plane of an image as a parametric curve

$$v(s) = (x(s), y(s))$$

- Contour is said to possess an energy (E_{snake}) which is defined as the sum of the three energy terms.

$$E_{snake} = E_{internal} + E_{external} + E_{constraint}$$

- The energy terms are defined so that the desired final position of the contour will have a minimum energy (E_{min})
- Therefore our problem of detecting objects reduces to an energy minimization problem.

What are these energy terms which do the trick for us??

Internal Energy (E_{int})

- Depends on the intrinsic properties of the curve.
- Sum of elastic energy and bending energy.

Elastic Energy ($E_{elastic}$):

- The curve is treated as an elastic rubber band possessing elastic potential energy.
- It discourages stretching by introducing tension.

$$E_{elastic} = \frac{1}{2} \int_s \alpha(s) |v_s|^2 ds \quad v_s = \frac{dv(s)}{ds}$$

- Weight $\alpha(s)$ allows us to control elastic energy along different parts of the contour. Considered to be constant α for many applications.
- Responsible for shrinking of the contour.

Bending Energy ($E_{bending}$):

- The snake is also considered to behave like a thin metal strip giving rise to bending energy.
- It is defined as sum of squared curvature of the contour.

$$E_{bending} = \frac{1}{2} \int_s \beta(s) |v_{ss}|^2 ds$$

- $\beta(s)$ plays a similar role to $\alpha(s)$.
- Bending energy is minimum for a circle – for a closed snake, or a line for an open one.
- Total internal energy of the snake can be defined as

$$E_{int} = E_{elastic} + E_{bending} = \int_s \frac{1}{2} (\alpha |v_s|^2 + \beta |v_{ss}|^2) ds$$

External energy of the contour (E_{ext})

- It is derived from the image.
- Define a function $E_{image}(x,y)$ so that it takes on its smaller values at the features of interest, such as boundaries.

$$E_{ext} = \int_s E_{image}(v(s)) ds$$

Key rests on defining $E_{image}(x,y)$. Some examples

- $E_{image}(x,y) = -|\nabla I(x,y)|^2$
- $E_{image}(x,y) = -|\nabla(G_c(x,y) * I(x,y))|^2$

Energy and force equations

- The problem at hand is to find a contour $v(s)$ that minimize the energy functional

$$E_{total} = \int_s \frac{1}{2} (\alpha(s) |v_s|^2 + \beta(s) |v_{ss}|^2) + E_{image}(v(s)) ds$$

- Using variational calculus and by applying Euler-Lagrange differential equation we get following equation

$$\alpha v_{ss} - \beta v_{ssss} - \nabla E_{image} = 0$$

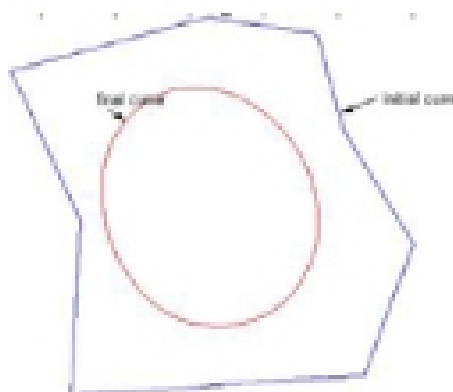
- Equation can be interpreted as a force balance equation.
- Each term corresponds to a force produced by the respective energy terms. The contour deforms under the action of these forces.

Elastic force

- Generated by elastic potential energy of the curve.

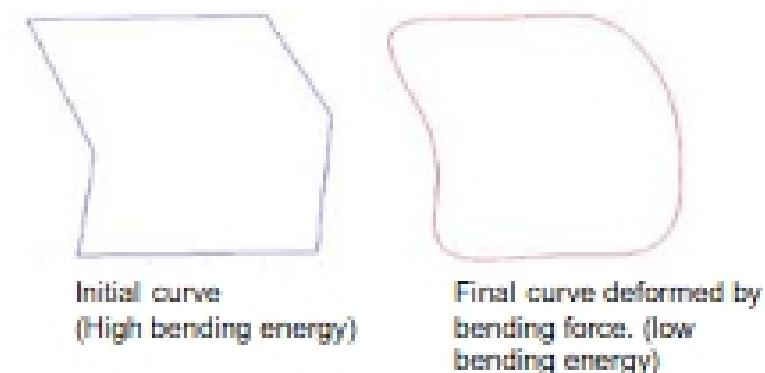
$$F_{elastic} = \alpha v_{ss}$$

- Characteristics (refer diagram)



Bending force

- Generated by the bending energy of the contour.
- Characteristics (refer diagram):

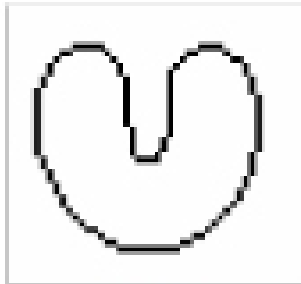


- Thus the bending energy tries to smooth out the curve.

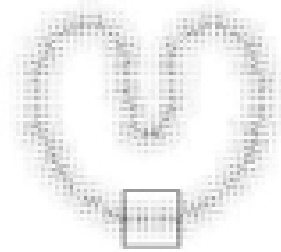
External force

$$F_{ext} = -\nabla E_{image}$$

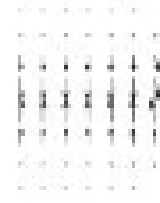
- It acts in the direction so as to minimize E_{ext}



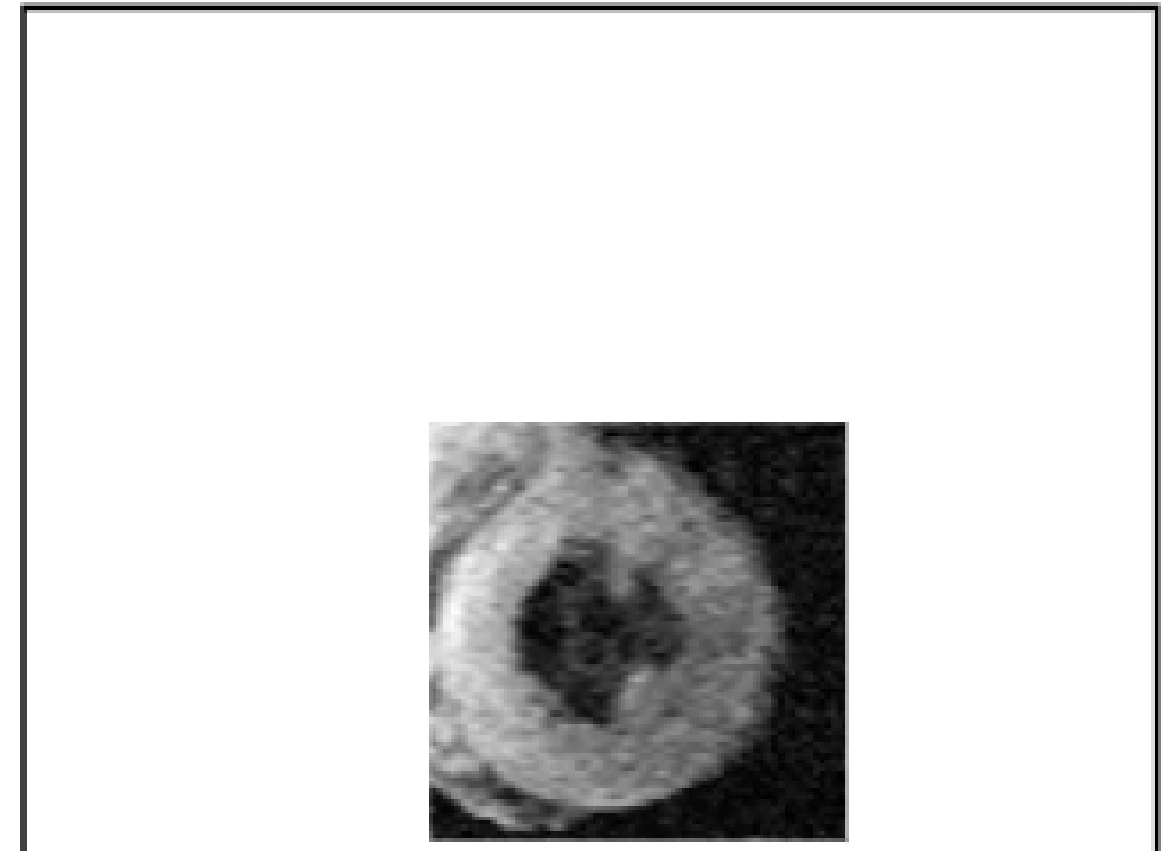
Image



External force



Zoomed in



Discretizing

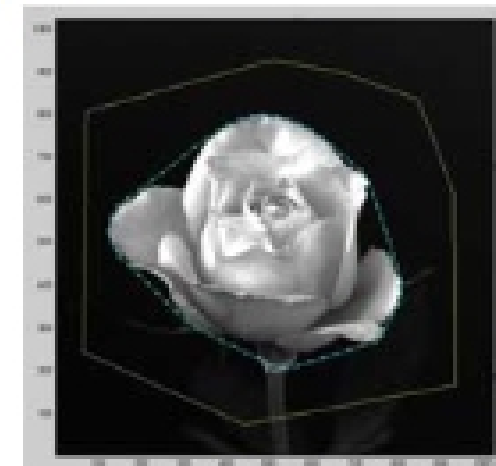
- the contour $v(s)$ is represented by a set of control points v_1, v_2, \dots, v_n
- The curve is piecewise linear obtained by joining each control point.
- Force equations applied to each control point separately.
- Each control point allowed to move freely under the influence of the forces.
- The energy and force terms are converted to discrete form with the derivatives substituted by finite differences.

Solution and Results

Method 1:

$$\alpha v_{xx} - \beta v_{xxxx} - \gamma \nabla E_{image} = 0$$

- γ is a constant to give separate control on external force.
- Solve iteratively.

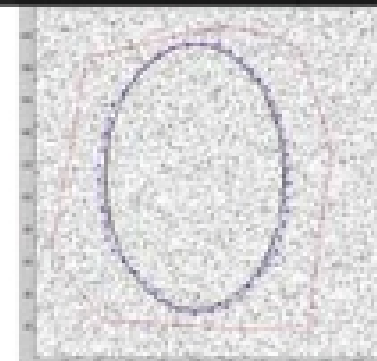


Method 2:

- Consider the snake to also be a function of time i.e. $v_i(s, t)$

$$\alpha v_{xx}(s, t) - \beta v_{xxxx}(s, t) - \nabla E_{image} = v_i(s, t) \quad v_i(s, t) = \frac{\partial v(s, t)}{\partial t}$$

- If RHS=0 we have reached the solution.
- On every iteration update control point only if new position has a lower external energy.
- Snakes are very sensitive to false local minima which leads to wrong convergence.



- Noisy image with many local minimas
- WGN sigma=0.1
- Threshold=15

