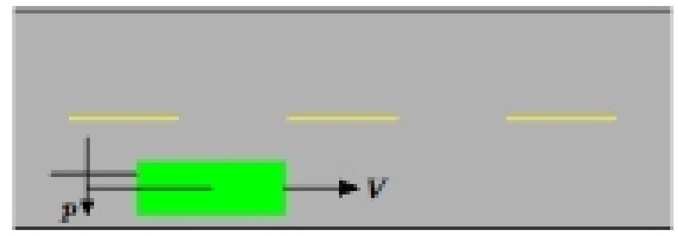
6.003: Signals and Systems

Feedback and Control

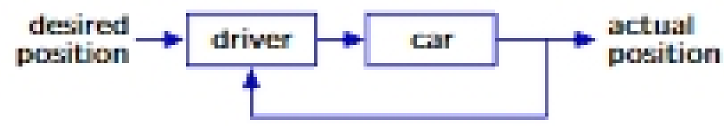
September 22, 2009

Feedback and Control

Feedback is pervasive in natural and artificial systems.

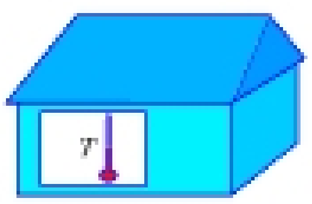


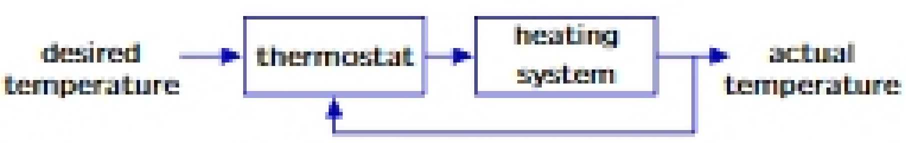
Turn steering wheel to stay centered in the lane.



Feedback and Control

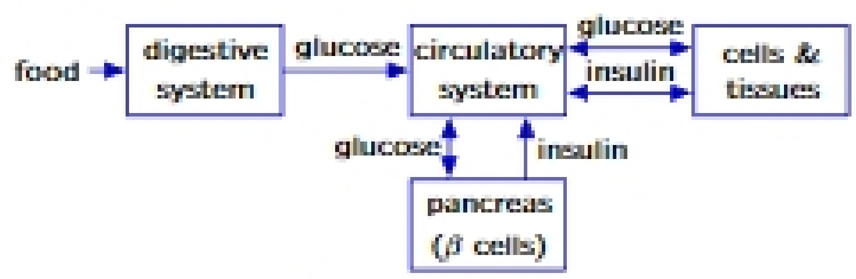
Feedback is useful for regulating a system's behavior, as when a thermostat regulates the temperature of a house.

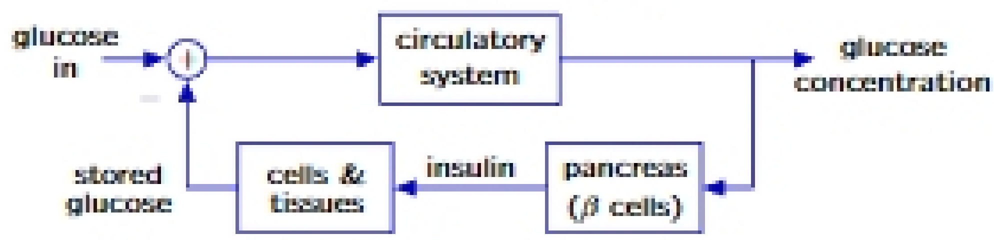




Feedback and Control

Concentration of glucose in blood is highly regulated and remains nearly constant despite episodic ingestion and use.

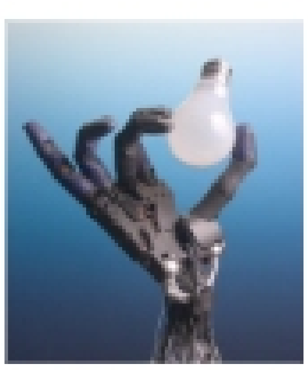




Feedback and Control

Motor control relies on feedback from pressure sensors in the skin as well as proprioceptors in muscles, tendons, and joints.

Try building a robotic hand to unscrew a lightbulb!



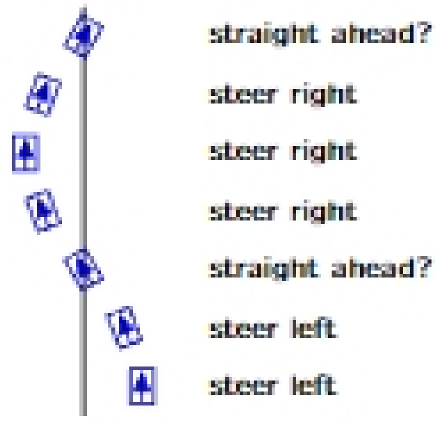
Shadow Dexterous Robot Hand (Wikipedia)

Today's goal

Use systems theory to gain insight into how to control a system.

Example: Steering a Car

Algorithm: steer left when car is right of center and vice versa.



Bad algorithm → poor performance.

Here we get persistent oscillations!

Outline of the lecture

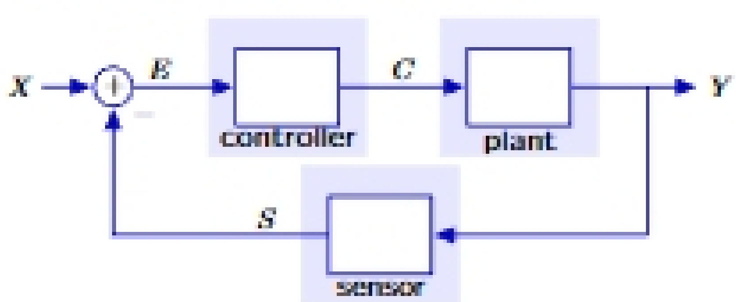
Understanding the structure of a control problem
automatic control → feedback

Analyzing feedback systems
feedback → cyclic paths → persistent outputs

Designing control systems
constructing well-behaved response properties

Structure of a Control Problem

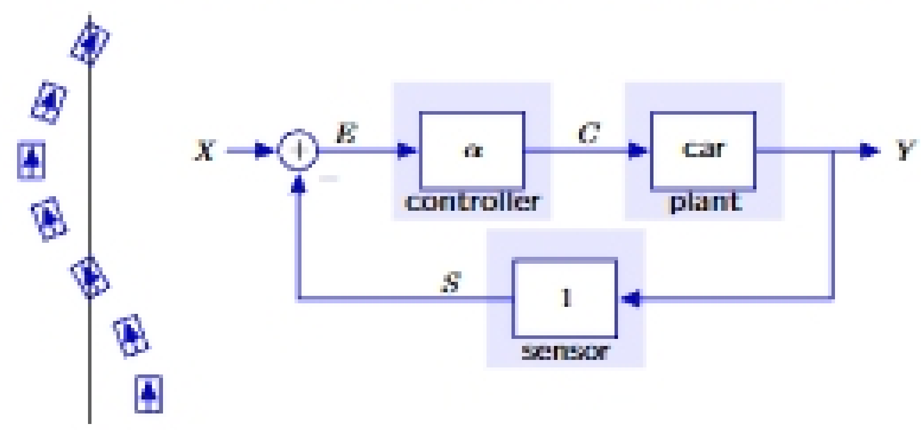
(Simple) Control systems have three parts.



The **plant** is the system to be controlled.
The **sensor** measures the output of the plant.
The **controller** specifies a command C to the plant based on the *difference* between the input X and sensor output S.

Three Parts in the Car Steering Problem

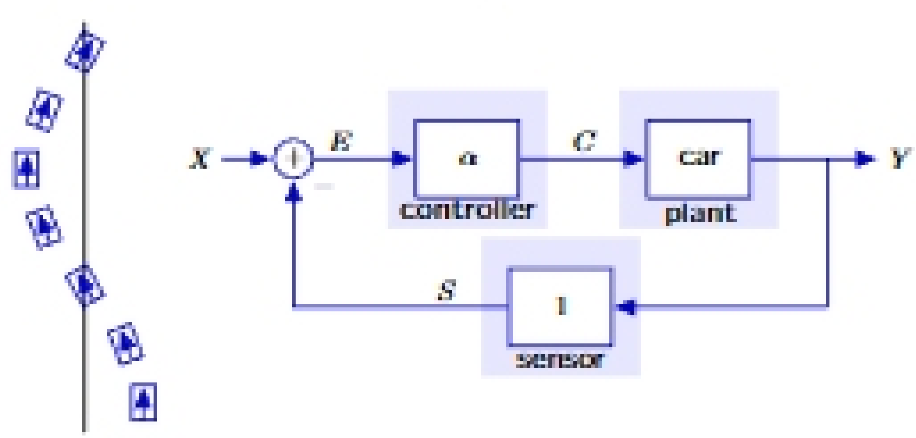
plant = car perfect sensor proportional controller



X = desired left/right position in lane ($=0$)
C = steering wheel position ($C = \alpha E = \alpha(X - S) = \alpha(X - Y)$)
Y = actual left/right position in lane

Car Steering Problem

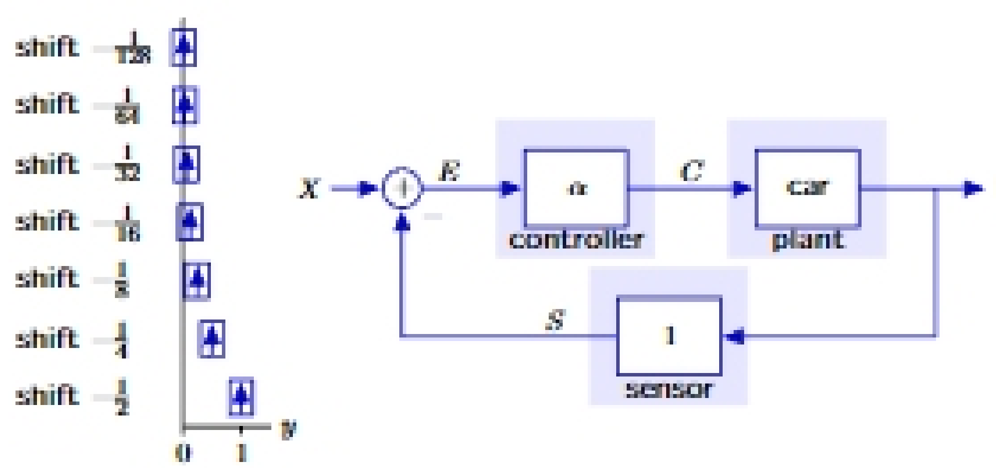
A realistic model for the car is complicated.



Turning the steering wheel rotates the car body so that forward motion of the vehicle changes the position in the lane.

Simpler Car Steering Problem

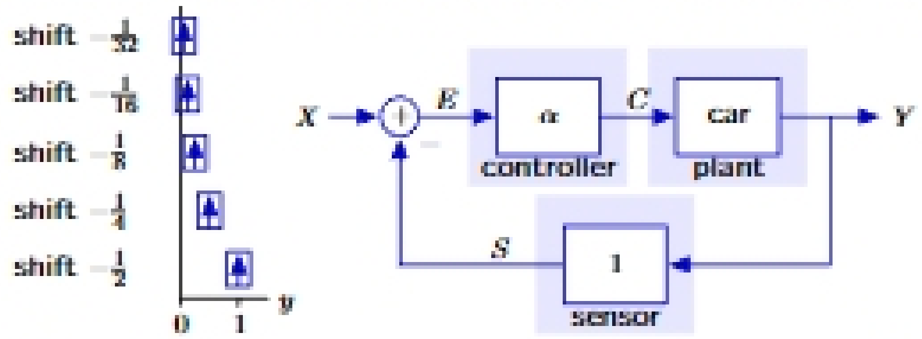
Start with a simpler problem: Imagine a (very non-physical) car that can instantly change its lateral position from $y[n]$ to $y[n] + c[n]$.



What is the value of α in this example?
What would be a better value of α?

Check Yourself

On each step, the car changes its position from $y[n]$ to $y[n]+c[n]$.

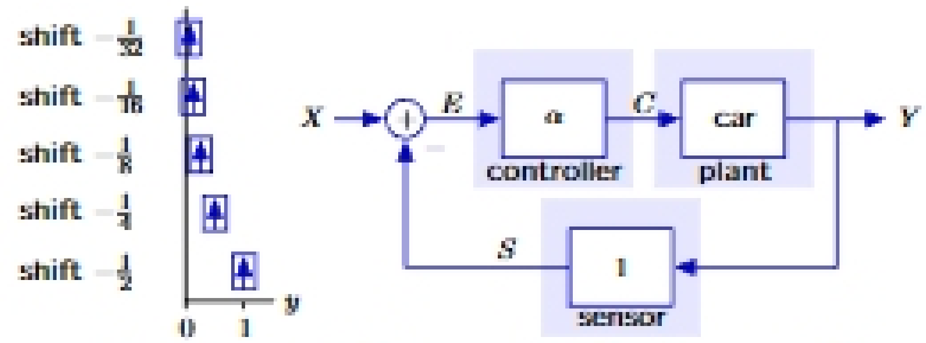


Find the appropriate model for this (very unusual) car.

1. $\mathcal{R}$ 2. $\frac{1}{\mathcal{R}}$ 3. $\frac{1}{1+\mathcal{R}}$ 4. $\frac{\mathcal{R}}{1-\mathcal{R}}$
5. none of the above

Check Yourself

What is the system functional $\frac{Y}{X}$ for the entire controller system?

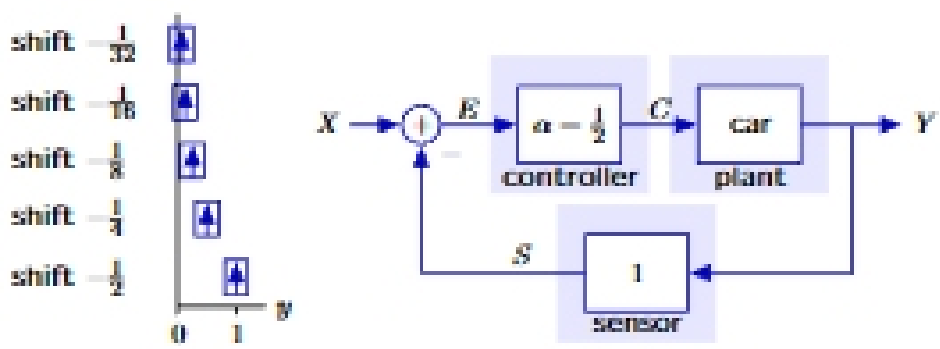


1. $\frac{\mathcal{R}}{1-\mathcal{R}}$ 2. $\alpha\frac{\mathcal{R}}{1-\mathcal{R}}$ 3. $\frac{\alpha\mathcal{R}}{1-(1-\alpha)\mathcal{R}}$ 4. $\frac{\alpha\mathcal{R}}{1-\alpha\mathcal{R}}$
5. none of the above

Designing a Controller

The "closed-loop" system has a single pole at $(1-\alpha)$.

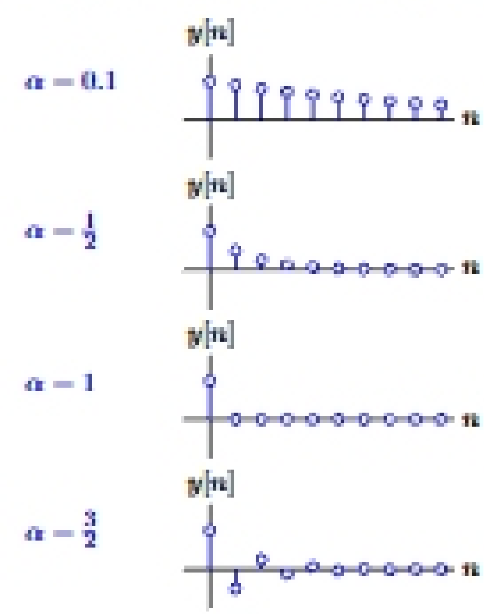
$$\frac{Y}{X}=\frac{\alpha\mathcal{R}}{1-(1-\alpha)\mathcal{R}}$$



Therefore, the output can deviate from the input for long periods of time, as we saw when $\alpha=\frac{1}{2}$.

Designing a Controller

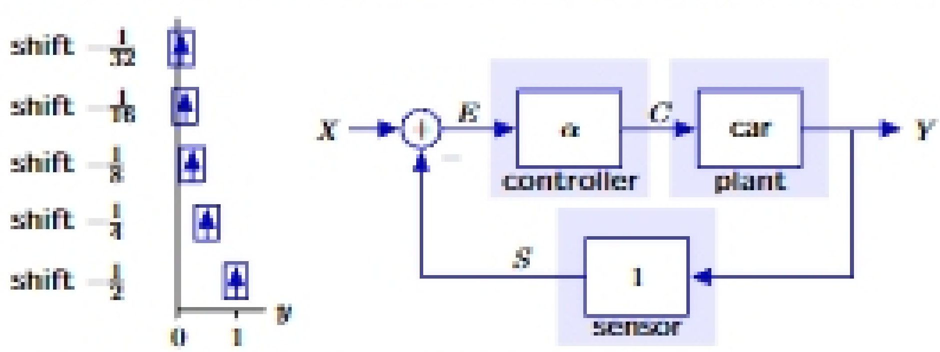
Range of responses $y[n]$ given $y[0]=1$ and $x[n]=0$.



Designing a Controller

Long-term deviations of the output from the input represent a failure to control the system in the desired fashion.

We would like the car to instantly move to the desired position.



Fortunately, we can control the shape of the output

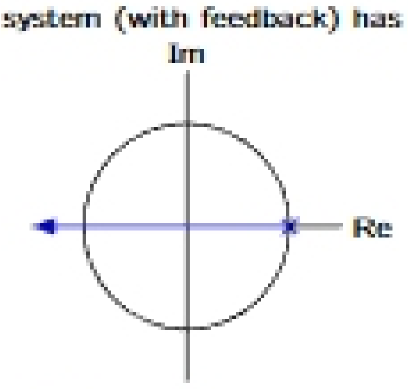
$$\frac{Y}{X}=\frac{\alpha\mathcal{R}}{1-(1-\alpha)\mathcal{R}}$$

by controlling the "closed-loop" pole at $(1-\alpha)$.

Designing a Controller

The "open-loop" system (car) has a pole at $z=1$ (an accumulator).

The "closed-loop" system (with feedback) has a pole at $z=1-\alpha$.



We can adjust α to optimize performance.

$$\frac{Y}{X}=\frac{\alpha\mathcal{R}}{1-(1-\alpha)\mathcal{R}}$$