

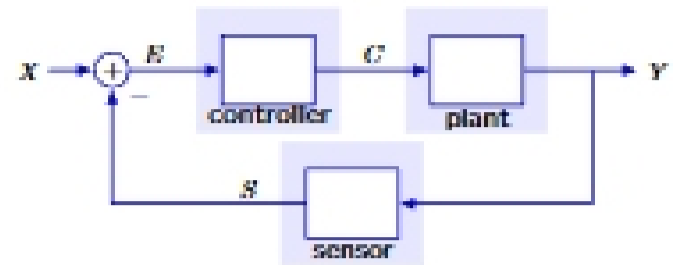
6.003: Signals and Systems

CT Feedback and Control

March 18, 2010

Feedback and Control

Feedback: simple, elegant, and robust framework for control.



We started with robotic driving.



Feedback and Control

Using feedback to enhance performance.

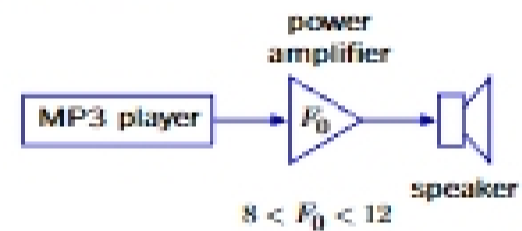
Examples:

- improve performance of an op amp circuit.
- control position of a motor.
- reduce sensitivity to unwanted parameter variation.
- reduce distortions.
- stabilize unstable systems
  - magnetic levitation
  - inverted pendulum

Feedback and Control

Reducing sensitivity to unwanted parameter variation.

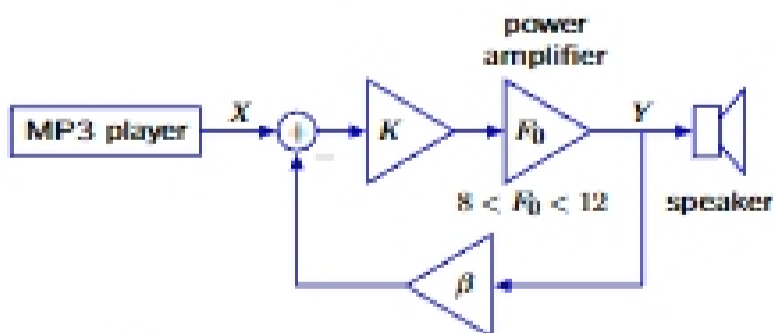
Example: power amplifier



Changes in  $F_0$  (due to changes in temperature, for example) lead to undesired changes in sound level.

Feedback and Control

Feedback can be used to compensate for parameter variation.



$$H(s) = \frac{KF_0}{1 + \beta KF_0}$$

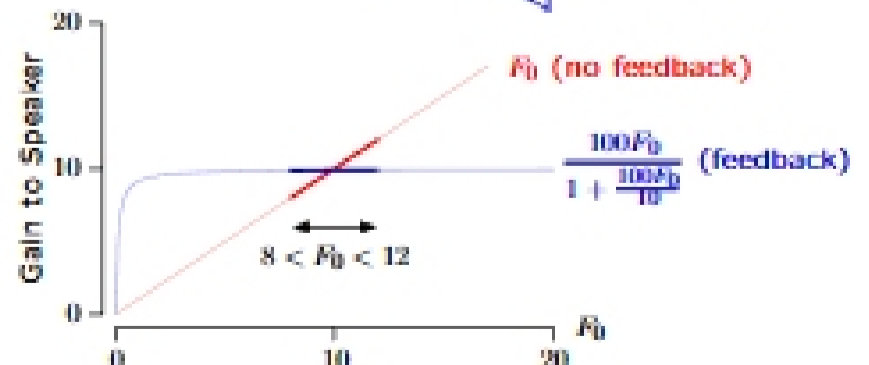
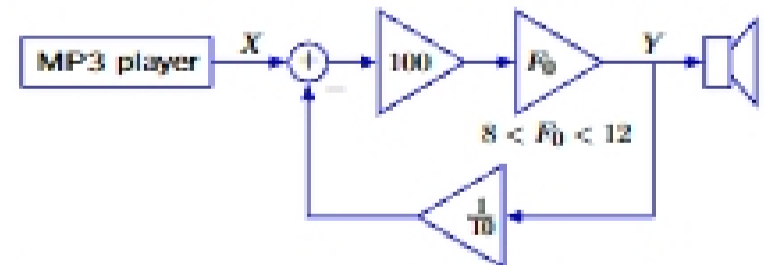
If  $K$  is made large, so that  $\beta KF_0 \gg 1$ , then

$$H(s) \approx \frac{1}{\beta}$$

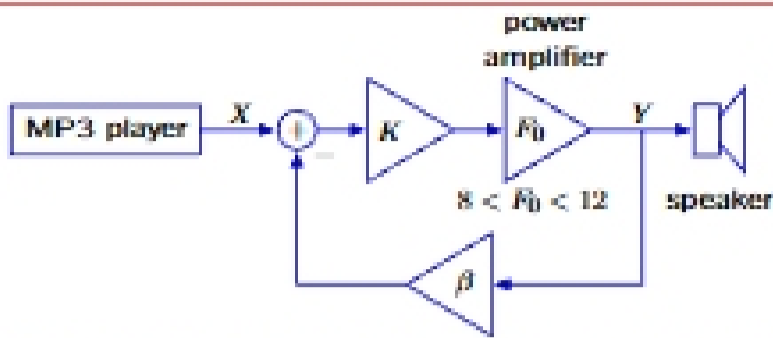
independent of  $K$  or  $F_0$ !

Feedback and Control

Feedback reduces the change in gain due to change in  $F_0$ .



**Check Yourself**



Feedback greatly reduces sensitivity to variations in  $K$  or  $R_0$ .

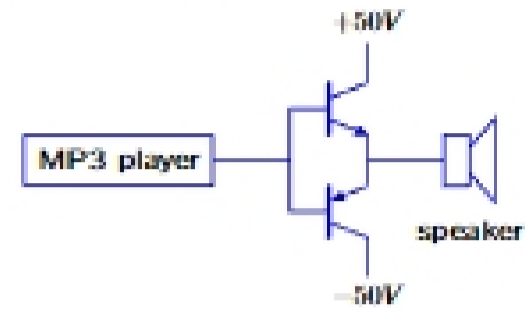
$$\lim_{K \rightarrow \infty} H(s) = \frac{KR_0}{1 + \beta KR_0} \rightarrow \frac{1}{\beta}$$

What about variations in  $\beta$ ? Aren't those important?

**Crossover Distortion**

Feedback can compensate for parameter variation even when the variation occurs rapidly.

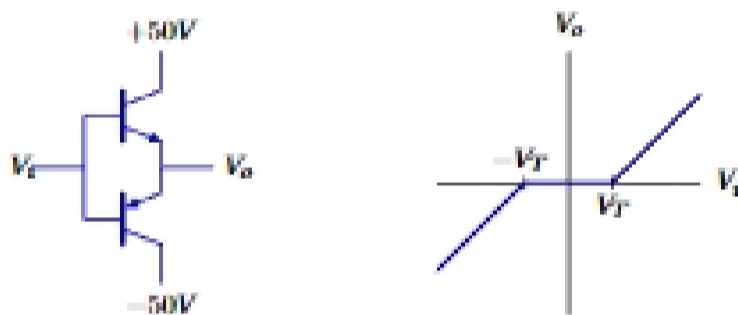
Example: using transistors to amplify power.



**Crossover Distortion**

This circuit introduces "crossover distortion."

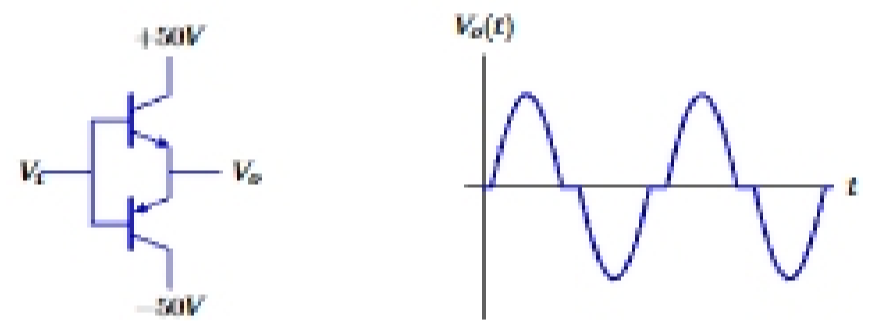
For the upper transistor to conduct,  $V_i - V_o > V_T$ .  
For the lower transistor to conduct,  $V_i - V_o < -V_T$ .



**Crossover Distortion**

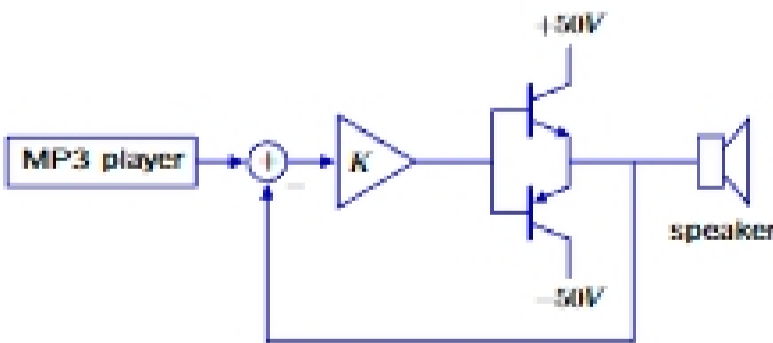
Crossover distortion can have dramatic effects.

Example: crossover distortion when the input is  $V_i(t) = B \sin(\omega_0 t)$ .



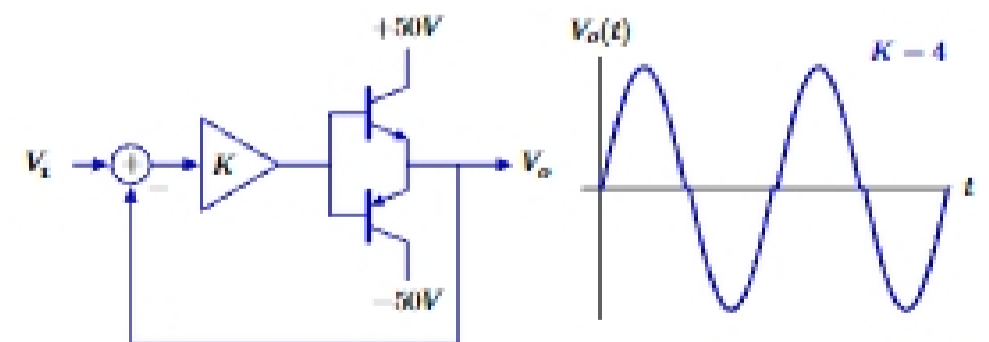
**Crossover Distortion**

Feedback can reduce the effects of crossover distortion.



**Crossover Distortion**

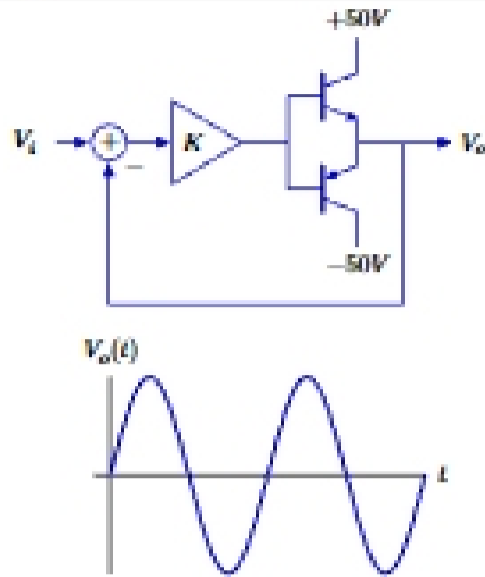
As  $K$  increases, feedback reduces crossover distortion.



**Crossover Distortion**

Demo

- original
- no feedback
- $K = 2$
- $K = 4$
- $K = 8$
- $K = 16$
- original



J.S. Bach, Sonata No. 1 in G minor Mvmt. IV. Presto  
Nathan Milstein, violin

**Feedback and Control**

Using feedback to enhance performance.

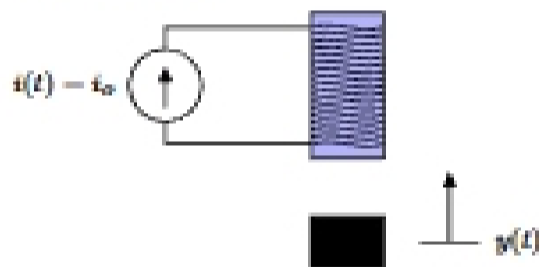
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- improve performance of an op amp circuit.
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**Control of Unstable Systems**

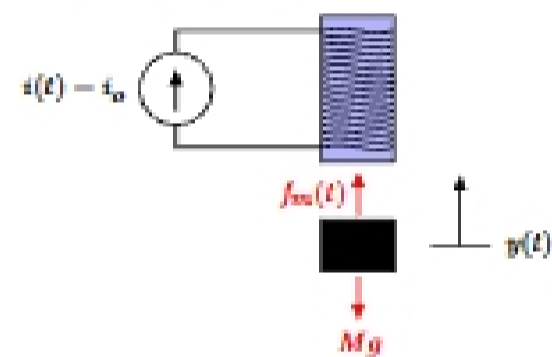
Feedback is useful for controlling **unstable** systems.

Example: Magnetic levitation.



**Control of Unstable Systems**

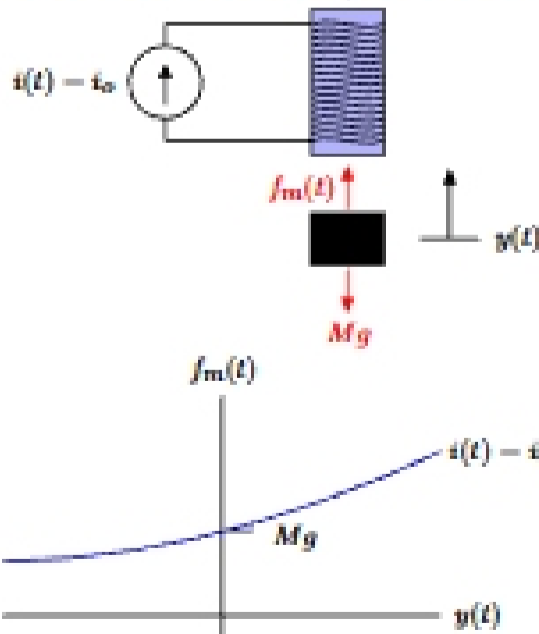
Magnetic levitation is unstable.



Equilibrium ( $y = 0$ ): magnetic force  $f_m(t)$  is equal to the weight  $Mg$ .  
 Increase  $y \rightarrow$  increased force  $\rightarrow$  further increases  $y$ .  
 Decrease  $y \rightarrow$  decreased force  $\rightarrow$  further decreases  $y$ .  
 Positive feedback!

**Modeling Magnetic Levitation**

The magnet generates a force that depends on the distance  $y(t)$ .



**Modeling Magnetic Levitation**

The net force accelerates the mass.

