ME451: Control Systems

Lecture 11
Routh-Hurwitz criterion: Control examples

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Course roadmap

Modeling
- ✔ Laplace transform
- ✔ Transfer function
- ✔ Models for systems
  - ✔ electrical
  - ✔ mechanical
  - ✔ electromechanical
- ✔ Block diagrams
- ✔ Linearization

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Analysis
- Time response
  - Transient
  - Steady state
- Frequency response
  - Bode plot
- ✔ Stability
  - Routh-Hurwitz
  - Nyquist

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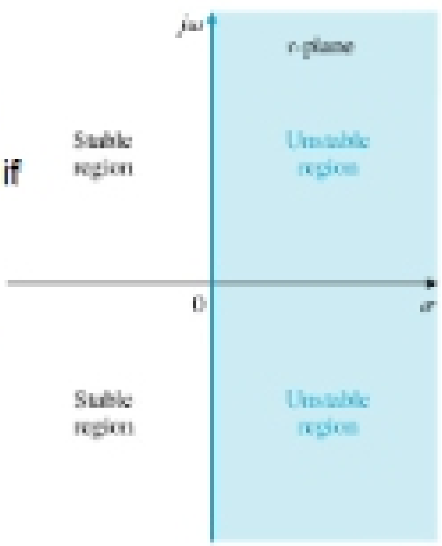
Design
- Design specs
- Root locus
- Frequency domain
- PID & Lead-lag
- Design examples

(Matlab simulations &) laboratories

Stability summary (review)

Let s_i be poles of rational G. Then, G is …

- (BIBO, asymptotically) stable if $\text{Re}(s_i)<0$ for all i.
- marginally stable if
  - $\text{Re}(s_i)\le 0$ for all i, and
  - simple root for $\text{Re}(s_i)=0$
- unstable if it is neither stable nor marginally stable.



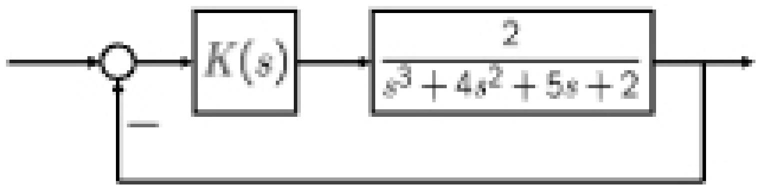
Routh-Hurwitz criterion (review)

$$Q(s) = a_n s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0$$

s^n	a_n	a_{n-2}	a_{n-4}	a_{n-6}	$\cdots$
s^{n-1}	a_{n-1}	a_{n-3}	a_{n-5}	a_{n-7}	$\cdots$
s^{n-2}	b_1	b_2	b_3	b_4	$\cdots$
s^{n-3}	c_1	c_2	c_3	c_4	$\cdots$
$\vdots$	$\vdots$	$\vdots$			
s^2	k_1	k_2			
s^1	l_1				
s^0	m_1				

The number of roots in the right half-plane is equal to the number of sign changes in the first column of Routh array.

Example 1



- Design K(s) that stabilizes the closed-loop system for the following cases.
  - K(s) = K (constant)
  - K(s) = K_P+K_I/s (PI (Proportional-Integral) controller)

Example 1: K(s)=K

- Characteristic equation

$$1 + K\frac{2}{s^3+4s^2+5s+2} = 0$$

$$\Rightarrow \quad s^3+4s^2+5s+2+2K = 0$$

- Routh array

s^3	1	5
s^2	4	$2+2K$
s^1	$\frac{18-2K}{4}$	
s^0	$2+2K$	

$$\Rightarrow \quad -1 < K < 9$$

Example 1: K(s)=K_P+K_I/s

- Characteristic equation

$$1 + \left(K_P + \frac{K_I}{s}\right)\frac{2}{s^3+4s^2+5s+2} = 0$$

$$\Rightarrow \quad s^4+4s^3+5s^2+(2+2K_P)s+2K_I = 0$$

- Routh array

s^4	1	5	$2K_I$
s^3	4	$2+2K_P$	
s^2	$\frac{18-2K_P}{4}$	$2K_I$	
s^1	$*$		
s^0	$2K_I$		

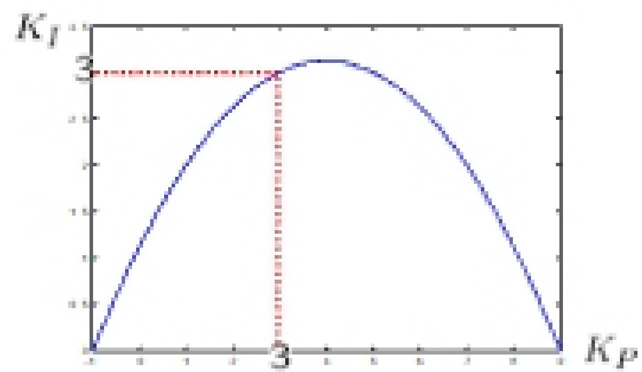
$$\frac{18-2K_P}{4} \rightarrow K_P < 9$$

$$2K_I \rightarrow K_I > 0$$

Example 1: Range of (K_P,K_I)

- From Routh array,

$$K_P < 9$$
$$K_I > 0$$
$$(1+K_P)(9-K_P) - 8K_I > 0$$



Example 1: K(s)=KP+KI/s (cont'd)

- Select KP=3 (<9)
- Routh array (cont'd)

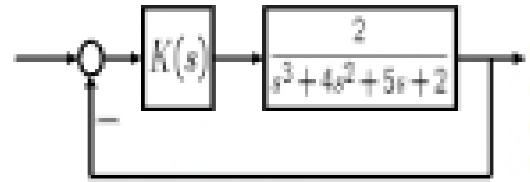
s^4	1	5	$2K_I$
s^3	4	8	
s^2	3	$2K_I$	
s^1	$\frac{24-8K_I}{3}$		
s^0	$2K_I$		

s^1, s^0 rows $\rightarrow 0 < K_I < 3$

- If we select different KP, the range of KI changes.

Example 1: What happens if KP=KI=3

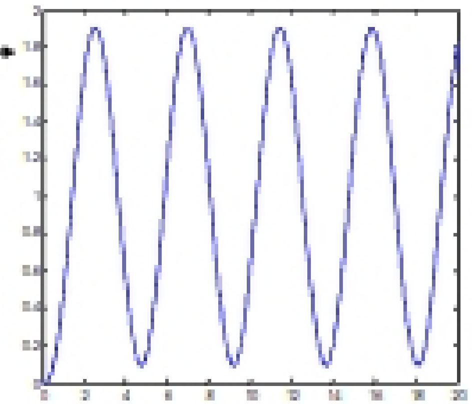
- Auxiliary equation $3s^2 + 6 = 0 \Leftrightarrow s = \pm\sqrt{2}j$



- Oscillation frequency

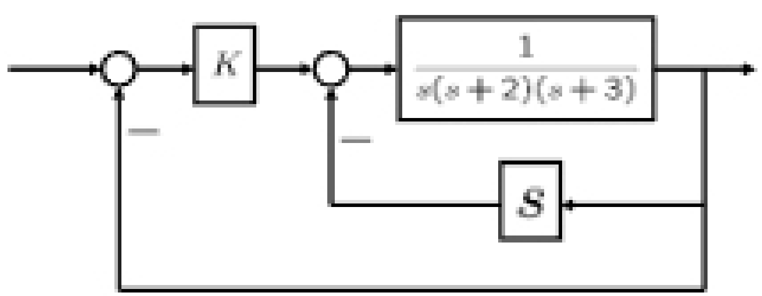
$\sqrt{2}$(rad/sec)

- Period $\frac{2\pi}{\sqrt{2}} \approx 4.4$(sec)



Unit step response

Example 2



- Determine the range of K and a that stabilize the closed-loop system.

Example 2 (cont'd)