8

Proportional and derivative control

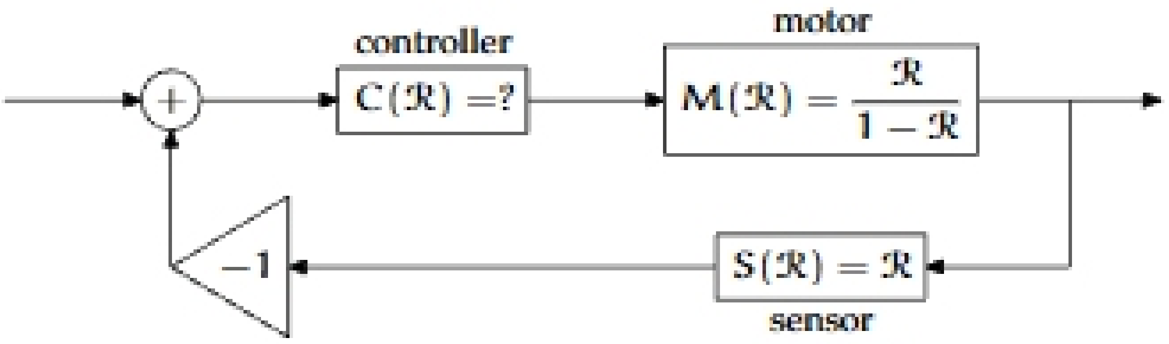
The goals of this chapter are:

- to introduce derivative control; and
- to study the combination of proportional and derivative control for taming systems with integration or inertia.

The controllers in the previous chapter had the same form: The control signal was a multiple of the error signal. This method cannot easily control an integrating system, such as the motor positioning a rod even without inertia. If the system has inertia, the limits of proportional control become even more apparent. This chapter introduces an alternative: derivative control.

8.1 Why derivative control

An alternative to proportional control is derivative control. It is motivated by the integration inherent in the motor system. We would like the feedback system to make the actual position be the desired position. In other words, it should copy the input signal to the output signal. We would even settle for a bit of delay on top of the copying. This arrangement is shown in the following block diagram:

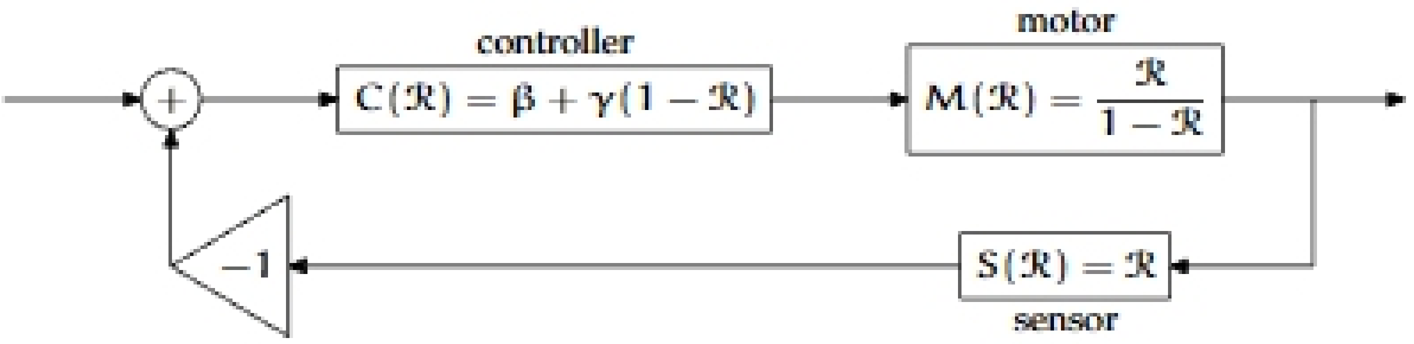


Since the motor has the functional $\mathcal{R}/(1-\mathcal{R})$, let's put a discrete-time derivative $1-\mathcal{R}$ into the controller to remove the $1-\mathcal{R}$ in the motor's denominator. With this **derivative control**, the forward-path cascade of the controller and motor contains only powers of $\mathcal{R}$. Although this method is too fragile to use alone, it is a useful idea. Pure derivative control is fragile because it uses pole–zero cancellation. This cancellation is mathematically plausible but, for the reasons explained in lecture, it produces unwanted offsets in the output. However, derivative control is still useful. As we will find, in combination with proportional control, it helps to stabilize integrating systems.

8.2 Mixing the two methods of control

Proportional control uses β as the controller. Derivative control uses $\gamma(1-\mathcal{R})$ as the controller. The linear mixture of the two methods is

$$C(\mathcal{R}) = \beta + \gamma(1-\mathcal{R}).$$



Let $F(\mathcal{R})$ be the functional for the entire feedback system. Its numerator is the forward path $C(\mathcal{R})M(\mathcal{R})$. Its denominator is $1-L(\mathcal{R})$, where $L(\mathcal{R})$ is the loop functional or **loop gain** that results from going once around the feedback loop. Here the loop functional is

$$L(\mathcal{R}) = -C(\mathcal{R})M(\mathcal{R})S(\mathcal{R}).$$

Don't forget the contribution of the inverting (gain$= -1$) element! So the overall system functional is

$$F(\mathcal{R}) = \frac{(\beta + \gamma(1-\mathcal{R}))\ \frac{\mathcal{R}}{1-\mathcal{R}}}{1 + (\beta + \gamma(1-\mathcal{R}))\ \frac{\mathcal{R}}{1-\mathcal{R}}\,\mathcal{R}}.$$

Clear the fractions to get

$$F(\mathcal{R}) = \frac{\text{whatever}}{1 - \mathcal{R} + (\beta + \gamma(1-\mathcal{R}))\mathcal{R}^2}.$$

The *whatever* indicates that we don't care what is in the numerator. It can contribute only zeros, whereas what we worry about are the poles. The poles arise from the denominator, so to avoid doing irrelevant algebra and to avoid cluttering up the expressions, we do not even compute the numerator as long as we know that the fractions are cleared.

The denominator is

$$1 - \mathcal{R} + (\beta + \gamma)\mathcal{R}^2 - \gamma\mathcal{R}^3.$$

This cubic polynomial produces three poles. Before studying their locations – a daunting task with a cubic – do an extreme-cases check: Take the limit $\gamma \to 0$ to turn off derivative control. The system should turn into the pure proportional-control system from the previous chapter. It does: The denominator becomes $1 - \mathcal{R} + \beta\mathcal{R}^2$, which is the denominator from Section 7.2. As the proportional gain β increases from 0 to ∞, the poles, which begin at 0 and 1, move inward; collide at $1/2$ when $\beta = 1/4$; then split upward and downward to infinity. Here is the root locus of this limiting case of $\gamma \to 0$, with only proportional control: