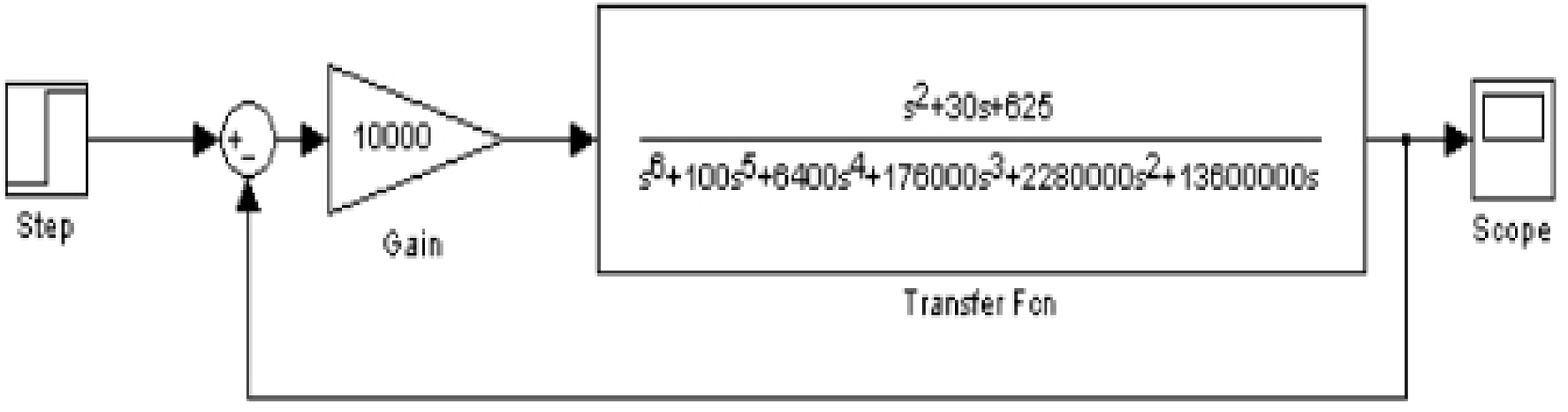
# Chapter 7: The Root Locus Method

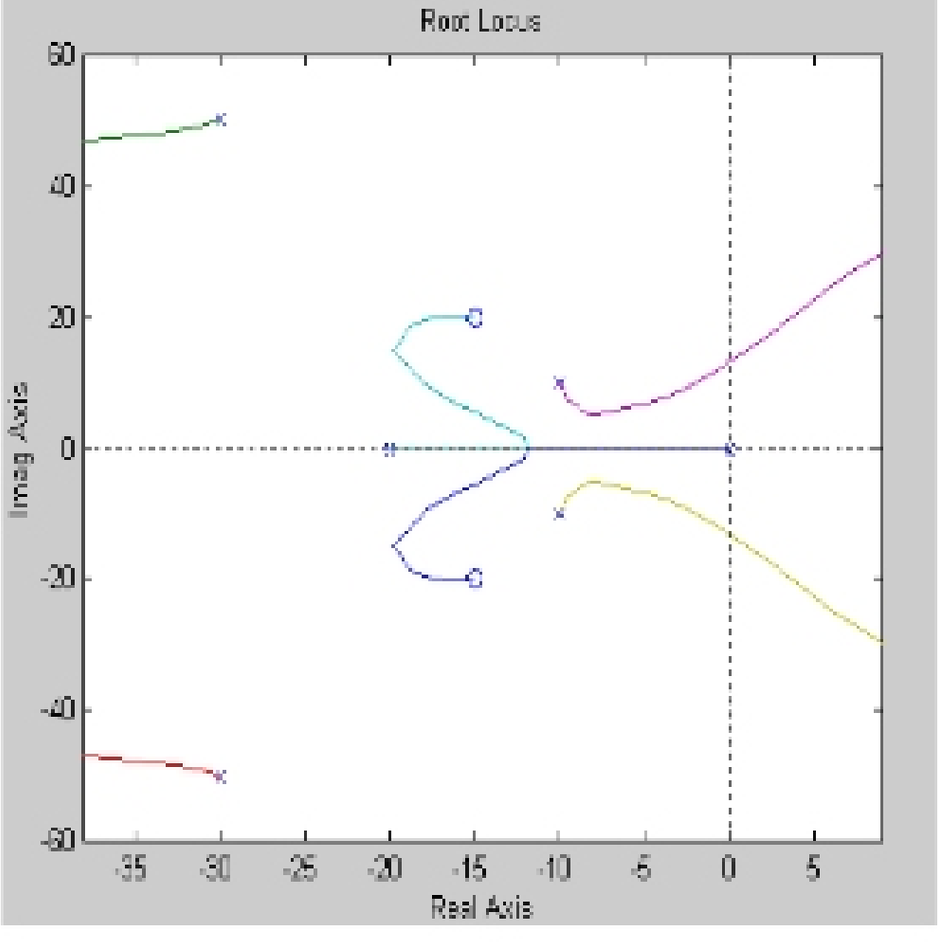
In the preceding chapters we discussed how the performance of a feedback system can be described in terms of the location of the roots of the characteristic equation in the s-plane. We know that the response of a closed-loop feedback system can be adjusted to achieve the desired performance by judicious selection of one or more system parameters. It is very useful to determine how the roots of the characteristic equation move around the s-plane as we change one parameter.

The locus of roots in the s-plane can be determined by a graphical method. A graph of the locus of roots as one system parameter varies is known as a root locus plot. The root locus is a powerful tool for designing and analyzing feedback control systems and is the main topic of this chapter. We will discuss practical techniques for obtaining a sketch of a root locus plot by hand. We also consider computer-generated root locus plots and illustrate their effectiveness in the design process. The popular PID controller is introduced as a practical controller structure.

We will show that it is possible to use root locus methods for design when two or three parameters vary. This provides us with the opportunity to design feedback systems with two or three adjustable parameters. For example the PID controller has three adjustable parameters. We will also define a measure of sensitivity of a specified root to a small incremental change in a system parameter.

# The Root Locus Method



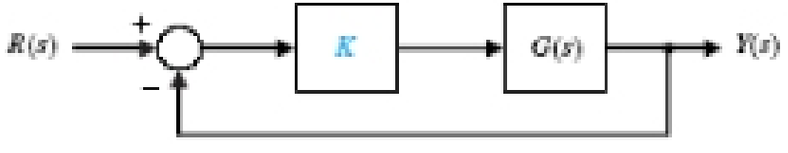



# The Root Locus Method

The root locus is a graphical procedure for determining the poles of a closed-loop system given the poles and zeros of a forward-loop system. Graphically, the locus is the set of paths in the complex plane traced by the closed-loop poles as the root locus gain is varied from zero to infinity.

In mathematical terms, given a forward-loop transfer function, $\mathbf{KG(s)}$
where K is the root locus gain, and the corresponding closed-loop transfer function

$$\frac{\mathbf{KG(s)}}{\mathbf{1 + KG(s)}}$$




the root locus is the set of paths traced by the roots of

$$\mathbf{1 + KG(s) = 0}$$

as K varies from zero to infinity. As K changes, the solution to this equation changes. This equation is called the characteristic equation. This equation defines where the poles will be located for any value of the root locus gain, K. In other words, it defines the characteristics of the system behavior for various values of controller gain.