ME451: Control Systems

Lecture 22
Frequency response

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Course roadmap

Modeling

- ✔ Laplace transform
- ✔ Transfer function
- ✔ Models for systems
  - ✔ electrical
  - ✔ mechanical
  - ✔ electromechanical
- ✔ Block diagrams
- ✔ Linearization

Analysis

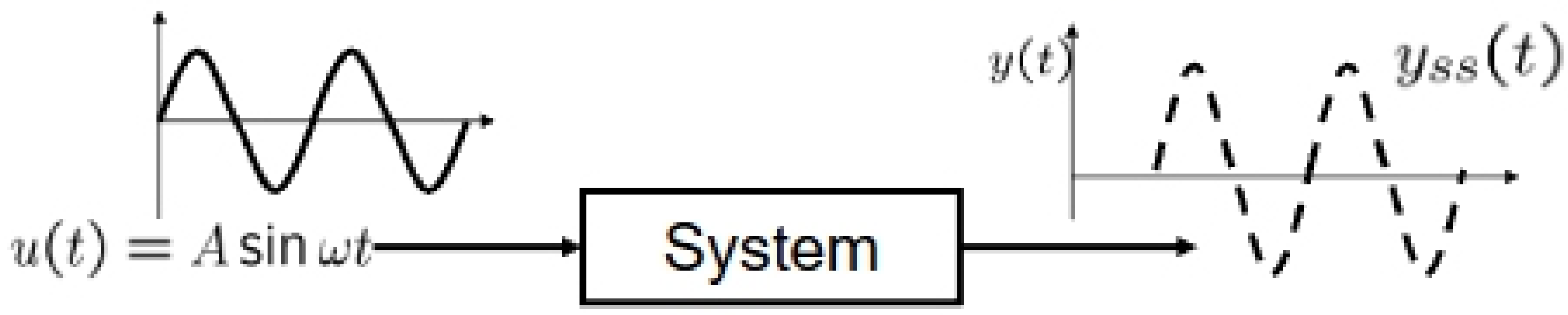
- ✔ Time response
  - ✔ Transient
  - ✔ Steady state
- Frequency response
  - (3) Bode plot
- ✔ Stability
  - ✔ Routh-Hurwitz
  - (3) Nyquist

Design

- (1) Design specs
- ✔ Root locus
- (3) Frequency domain
- (1) ✔ PID & Lead-lag
- (2) Design examples

(Matlab simulations &) laboratories

What is frequency response?

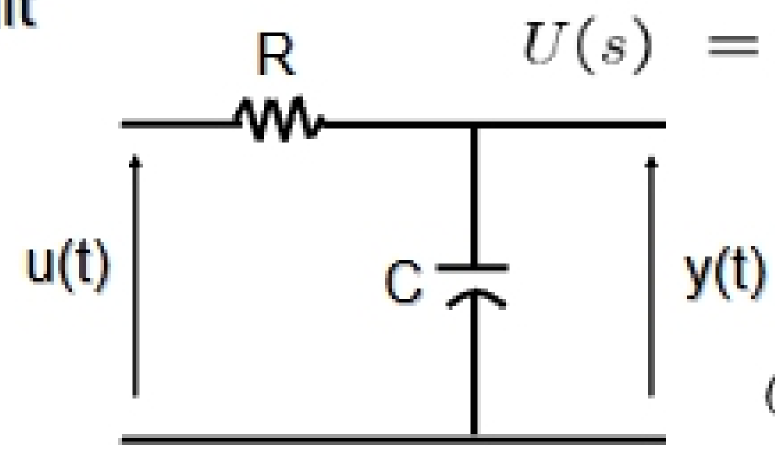


- We would like to analyze a system property by applying a *test sinusoidal input* u(t) and observing a response y(t).
- Steady state response $y_{ss}(t)$ (after transient dies out) of a system to sinusoidal inputs is called ***frequency response***.

A simple example

- RC circuit

$$\begin{aligned} Y(s) &= \tfrac{1}{sC} I(s) \\ U(s) &= (R + \tfrac{1}{sC}) I(s) \end{aligned}$$



$$G(s) = \frac{1}{RCs + 1}$$

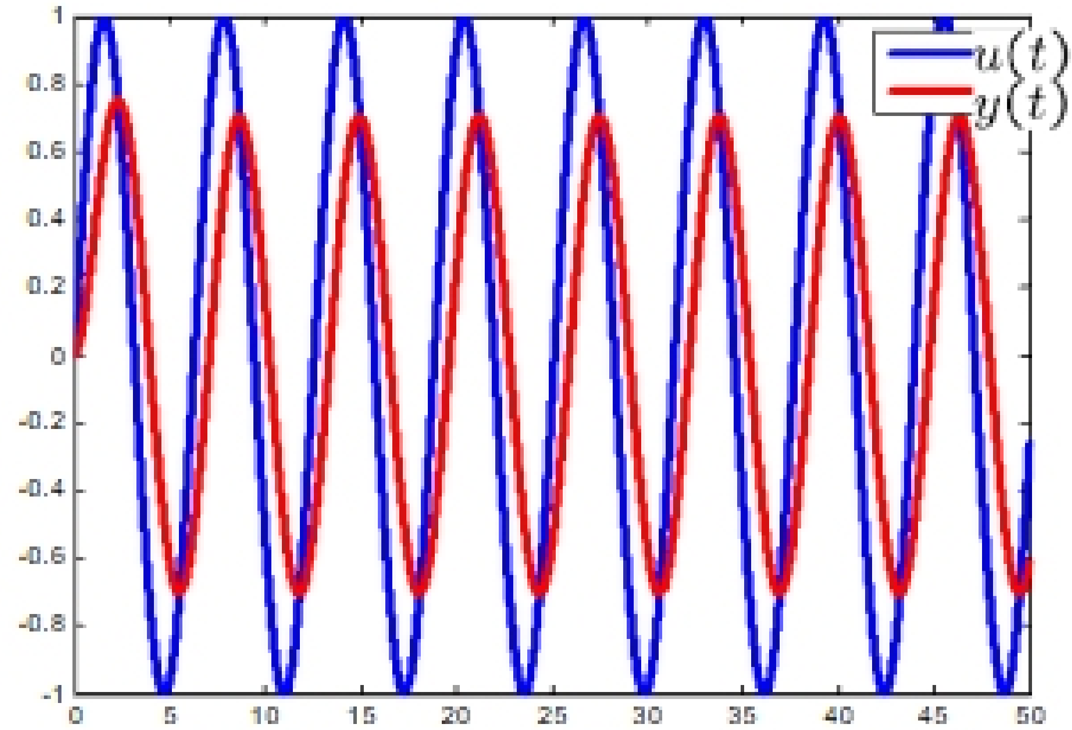
- Input a sinusoidal voltage u(t)
- What is the output voltage y(t)?

An example (cont'd)

- TF (R=C=1)

$$G(s) = \frac{1}{s+1}$$

- u(t)=sin(t)



At steady-state, u(t) and y(t) has same frequency, but different amplitude and phase!

An example (cont'd)

- Derivation of y(t)

$$Y(s) = G(s)U(s) = \frac{1}{s+1} \cdot \frac{1}{s^2+1} = \frac{1}{2}\left(\frac{1}{s+1} + \frac{-s+1}{s^2+1}\right)$$

Partial fraction expansion

- Inverse Laplace

$$y(t) = \frac{1}{2}\left(e^{-t} - \cos t + \sin t\right)$$

0 as t goes to infinity.

$$y_{ss}(t) = \frac{1}{2}(-\cos t + \sin t) = \frac{1}{\sqrt{2}}\sin(t - 45^\circ)$$

(Derivation for general G(s) is given at the end of lecture slide.)