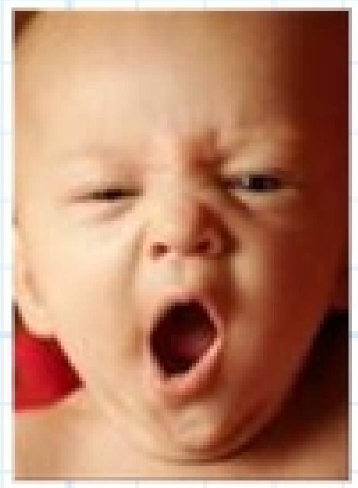
The Directional Coupler

A **directional coupler** is a 4-port network that is designed to **divide** and **distribute** power.



Although this would seem to be a particularly **mundane** and simple task, these devices are both very **important** in microwave systems, and very **difficult** to design and construct.

Two of the **reasons** for this difficulty are our desire for the device to be:

1. Matched

B. Lossless

Thus, we require a **matched**, **lossless**, and (to make it simple) **reciprocal** 4-port device!

Recall that a matched, lossless, reciprocal, 4-port device was difficult to even **mathematically** determine, as the resulting scattering matrix must be (among other things) **unitary**.

However, we were able to determine two possible mathematical solutions, which we called the **symmetric** solution:

$$\mathcal{S} = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

And the **asymmetric** solution:

$$\mathcal{S} = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

wherein for both cases, the relationship:

$$|\alpha|^2 + |\beta|^2 = 1$$

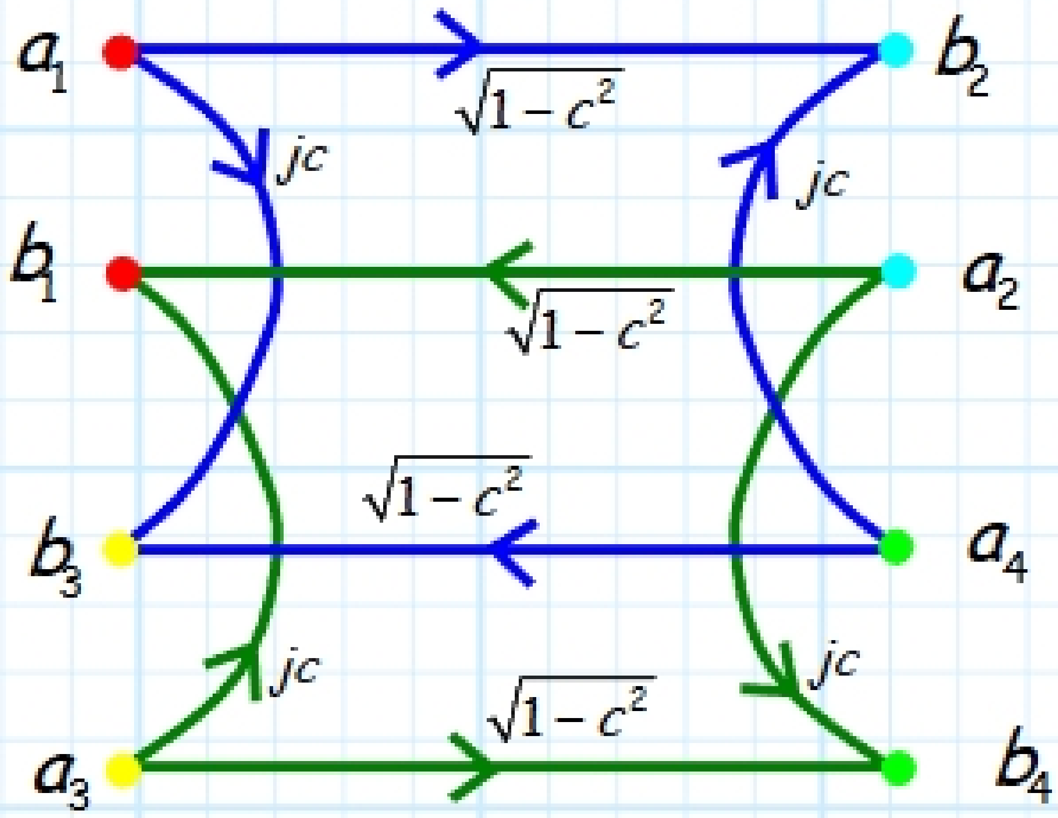
must be true in order for the device to be **lossless** (i.e., for $\mathcal{S}$ to be unitary).

For most couplers we will find that α and β can (at least ideally) be represented by a real value c, known as the **coupling coefficient**.

$$\beta = c \qquad\qquad \alpha = \sqrt{1-c^2}$$

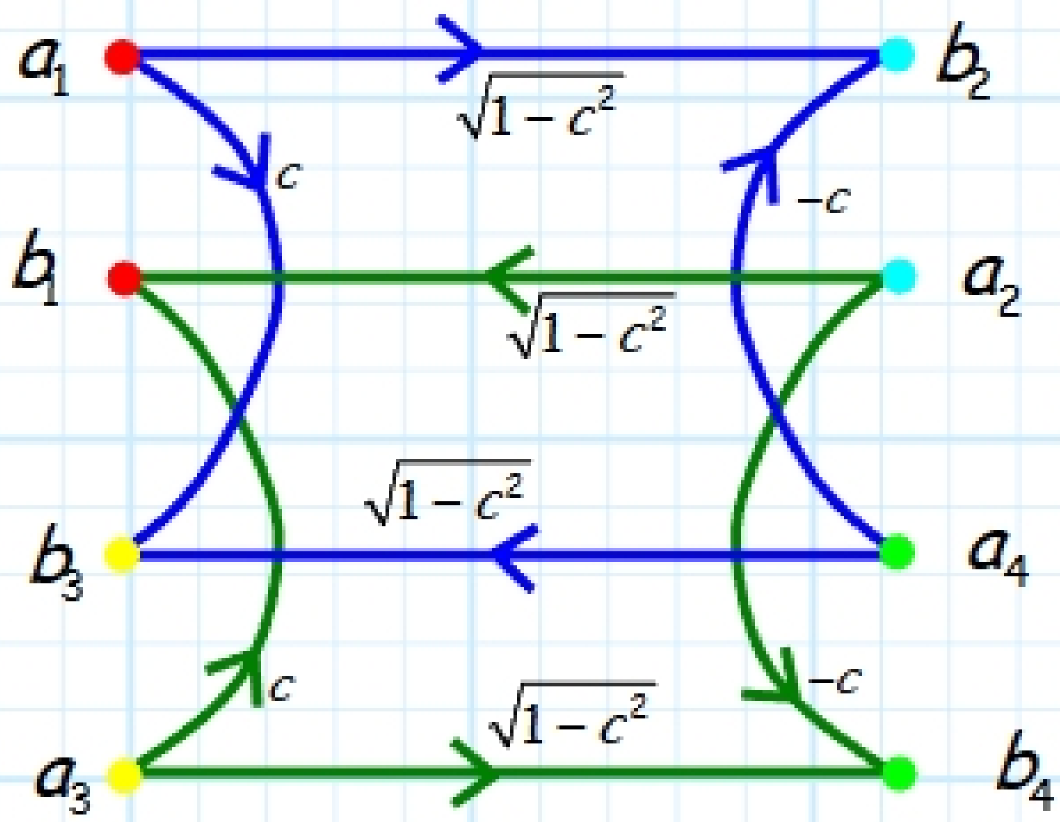
The **symmetric** solution is thus described as:

$$\mathcal{S} = \begin{bmatrix} 0 & \sqrt{1-c^2} & jc & 0 \\ \sqrt{1-c^2} & 0 & 0 & jc \\ jc & 0 & 0 & \sqrt{1-c^2} \\ 0 & jc & \sqrt{1-c^2} & 0 \end{bmatrix}$$



And the **asymmetric** solution is:

$$\mathcal{S} = \begin{bmatrix} 0 & \sqrt{1-c^2} & c & 0 \\ \sqrt{1-c^2} & 0 & 0 & -c \\ c & 0 & 0 & \sqrt{1-c^2} \\ 0 & -c & \sqrt{1-c^2} & 0 \end{bmatrix}$$



Additionally, for a directional coupler, the coupling coefficient c will be less than $1/\sqrt{2}$ **always**. Therefore, we find that:

$$0 \le c \le \frac{1}{\sqrt{2}} \quad \text{and} \quad \frac{1}{\sqrt{2}} \le \sqrt{1-c^2} \le 1$$

Let's see what this means in terms of the **physical behavior** of a directional coupler. First, consider the case where some signal is incident on **port 1**, with power P_1^+.