Math 128 Final Exam Spring '10

You may use a (non-programmable) scientific calculator and a 3 × 5 note card for the exam. This exam has 15 questions worth 5 points each. The maximum possible score is 105.

1. The equation

$$z^2 + 2yz^3 + x^2 z = 4 \qquad (*)$$

defines z implicitly as a function of x and y. Find the value at the point $(1, 1, 1)$ of the partial derivative

$$\frac{\partial z}{\partial y}$$

- **a.** 1/3
- **b.** 1/9
- **c.** 2/3
- **d.** 2/9
- **e.** $\boxed{-2/9 *}$
- **f.** −2/3
- **g.** −1/9
- **h.** −1/3
- **i.** 0
- **j.** 1

Differentiate $(*)$ with respect to y then set $x = y = z = 1$

$$\frac{\partial}{\partial y}(*) = 2z\frac{\partial z}{\partial y} + 2z^3 + 2y\,3z^2\frac{\partial z}{\partial y} + x^2\frac{\partial z}{\partial y}$$

& $\frac{\partial}{\partial y}$ of right hand side of $(*)$ is 0

so $0 = 2\frac{\partial z}{\partial y} + 2 + 6\frac{\partial z}{\partial y} + \frac{\partial z}{\partial y}$

$$9\frac{\partial z}{\partial y} = -2$$

Alternatively – Formula (1) pg 417

2. For

$$z = x^2 y^3 \text{ and } x = 3t - s,\ y = \frac{st^2}{2}$$

compute $\frac{\partial z}{\partial t}$ when $s = 2, t = 1$.

- **a.** 56
- **b.** $\boxed{12 *}$
- **c.** 44
- **d.** 88
- **e.** 16
- **f.** 60
- **g.** 40
- **h.** 32
- **i.** 84
- **j.** 28

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t}$$

$$= 2xy^3(3) + 3x^2y^2 \cdot \frac{2st}{2}$$

$\left.\begin{matrix} s = 2 \\ t = 1 \end{matrix}\right\} \rightarrow \begin{matrix} x = 1 \\ y = 1 \end{matrix}$

so, subtituting

Answer $= 2 \cdot 3 + 3 \cdot 2 = 12$

3. Find the linearization of (linear approximation to) the function

$$f(x,y) = \frac{1}{1+2x+xy-y^2}$$

at the point $(x,y) = (0,0)$

a. 1
b. $1+2x$
c. $1+2x+xy$
d. $1+2x+xy-y^2$
e. $\boxed{1-2x *}$
f. $1-2x-xy$
g. $1-2x-xy+y^2$
h. $1-xy+y^2$
i. $1+xy-y^2$
j. $1-y^2$

Answer $= f(0,0) + \frac{\partial f}{\partial x}(0,0)\,x + \frac{\partial f}{\partial y}(0,0)\,y$

$f_x = \frac{-1}{(\ )^2}(2+y) = -2$ at $(0,0)$

$f_y = \frac{-1}{(\ )^2}(x-2y) = 0$ at $(0,0)$

$f(0,0) = 1$

Ans $= 1-2x$

4. Find the equation of the plane tangent to the graph of the function $f(x,y) = x^2 + 2xy^2$ at the point corresponding to $x = 1$, $y = 2$.

a. $-10x+8y-z = 13$
b. $x+10y-8z = 15$
c. $x-10y-8z = 10$
d. $\boxed{10x+8y-z = 17 *}$
e. $10x+y-8z = -8$
f. $x+10y-8z = -13$
g. $x+8y-10z = -10$
h. $10x+y-8z = -15$
i. $x+10y-8z = 0$
j. $x+8y-10z = 22$

at $x=1$, $y=2$, ~~z=~~

we have $z = f(x,y) = 1+2\cdot1\cdot4 = 9$

and $\frac{\partial f}{\partial x} = 2x + 2y^2 \longrightarrow 10$

$\frac{\partial f}{\partial y} = 4xy \longrightarrow 8$

Ans : $z - 9 = 10(x-1) + 8(y-2)$

so $10x + 8y - z = 17$

5. The function $f(x,y) = 3xy - x^3 - y^3$. has two critical points. One is at the origin. Locate the second one and classify both of them. The types of the critical points at the origin and then at the other critical point are:

a. Saddle point, local min.
b. Saddle point, local max*.
c. local max, local min.
d. local min, local max.
e. local min, local min.
f. local max, local max.
g. test fails, local min.
h. Saddle point, saddle point.
i. local max, saddle point
j. local min, saddle point.

$f_x = 3y - 3x^2$
$f_y = 3x - 3y^2$

for both to equal 0 we must have
$y = x^2$
$x = y^2$
substitute 1st into 2nd
$x = x^4$ so $x - x^4 = 0$
so $x(1-x^3) = 0$ so $x = 0$ or $x = 1$

$y = x^2$ so pts are $(0,0)$ + $(1,1)$

The Hessian is $\begin{pmatrix} -6x & 3 \\ 3 & -6x \end{pmatrix}$

which at the two points is $\begin{pmatrix} 0 & 3 \\ 3 & 0 \end{pmatrix}$ + $\begin{pmatrix} -6 & 3 \\ 3 & -6 \end{pmatrix}$

so Ans is B

6. The function

$$f(x,y) = 4x^{1/2}y^{1/4} - 2x - y$$

has a maximum in the region $x \geq 0$, $y \geq 0$. Find the coordinates of the point at which the maximum occurs.

a. $(1,2)$
b. $(2,1)$
c. $(1,1)$ *
d. $(1,\sqrt{2})$
e. $(\sqrt{2},1)$
f. $(\sqrt{2},\sqrt{2})$
g. $(2,\sqrt{2})$
h. $(\sqrt{2},2)$
i. $(2,1/2)$
j. $(\sqrt{2},1/2)$

$0 = f_x = 4\cdot\frac{1}{2}x^{-1/2}y^{1/4} - 2$
$0 = f_y = 4\cdot\frac{1}{4}x^{1/2}y^{-3/4} - 1$

1st eqn gives $1 = x^{-1/2}y^{1/4}$
2nd $1 = x^{1/2}y^{-3/4}$

so $x^{-1/2}y^{1/4} = x^{1/2}y^{-3/4}$

so $x = y$

back to 1st equation with $x = y$
$0 = 4\cdot\frac{1}{2}x^{-1/2}x^{1/4} - 2$

so $1 = x^{-1/4}$

$\left.\begin{array}{l} \text{so } x = 1 \\ + \ x = y \end{array}\right\}$ so $(1,1)$