3D Transformations

CSS 442/542

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Homogeneous Coordinates

- The point (x, y, z) is represented as a column-vector with an added $w \neq 0$ coordinate:

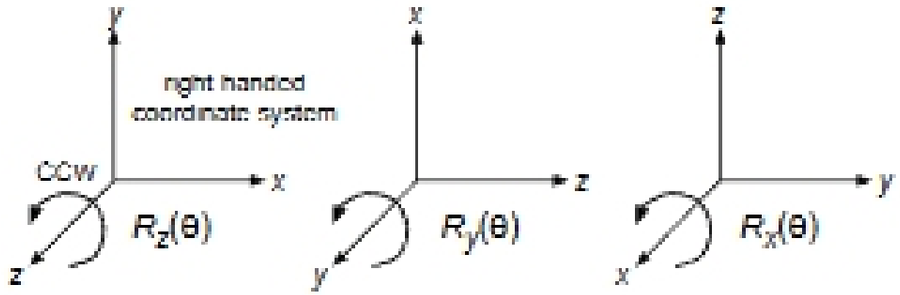
$$\begin{bmatrix} xw \\ yw \\ zw \\ w \end{bmatrix} \equiv \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \mapsto (x, y, z)$$

- 3-D *affine transformations* are represented with a 4×1 matrix A of the following form:

$$\begin{bmatrix} A_{00} & A_{01} & A_{02} & A_{03} \\ A_{10} & A_{11} & A_{12} & A_{13} \\ A_{20} & A_{21} & A_{22} & A_{23} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \mapsto \begin{bmatrix} x' \\ y' \\ z' \\ w \end{bmatrix}$$

- We use $w = 0$ for *direction vectors* $\vec{v} = (x, y, z)$. Effectively ignores "translation" component of A.

Rotation about principal axes



$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$