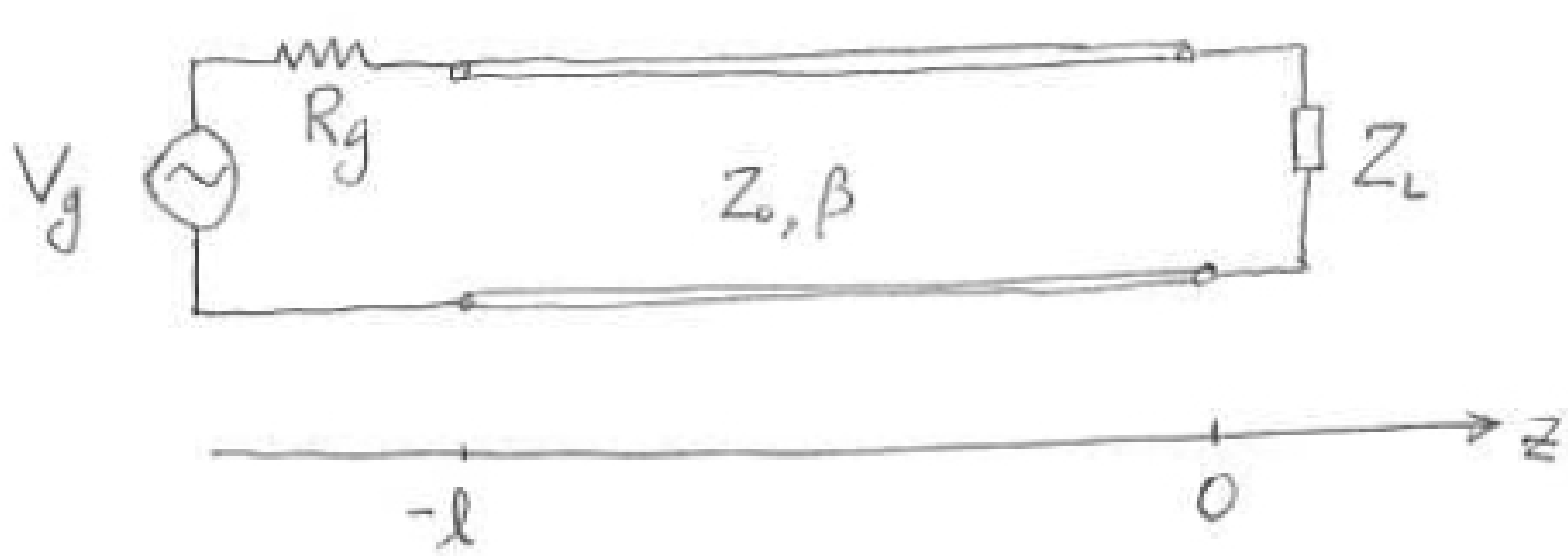
The following problems refer to this graph



$V_g = 120V$
$R_g = 30\Omega$
$Z_o = 50\Omega$ —— characteristic impedance of transmission line
$\beta = \frac{2\pi}{\lambda}$, $\lambda = 8$ cm
$l = 26.64$
$Z_L = 40 - j70\Omega$

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1. Calculate Γ_L, Φ_L, VSWR, z_{vm}

Γ_L = Load reflection coefficient $\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{40-j70-50}{40-j70+50} = 0.31-j0.54$

$= |\Gamma_L| e^{j\Phi_L} = 0.62 e^{-j60.26°}$

$\Phi_L = -60.26°$

VSWR = Voltage Standing wave ratio

$= \frac{1+|\Gamma_L|}{1-|\Gamma_L|} = \frac{1+0.62}{1-0.62} = 4.26$

z_{vm} = distance of the first voltage min position to Load

The phase of reflection coefficient

is $\Phi = \Phi_L + 2\beta z = \Phi_L - 2\beta|z|$

when $\Phi = \pm\pi, \pm 3\pi, \ldots$

Since $\Phi_L = -60.26° = -1.05$ (Rad) < 0

Hence: $-1.05 - 2\beta(z_{vm}) = -\pi$

Note: z_{vm} is a positive distance

$$-1.05 - 2\cdot\frac{2\pi}{0.08} z_{vm} = -3.14 \Rightarrow 0.0267\,\text{m}$$

or 2.67 cm

2. Calculate the input impedance $Z_{in}(-l)$ and the reflection coefficient $\Gamma(-l)$

$$\Gamma(z) = \Gamma_L e^{j2\beta z} = |\Gamma_L| e^{j\Phi_L - j2\beta|z|}$$

$$\Gamma(-l) = |\Gamma_L| e^{j\Phi_L - j2\beta l} = 0.62\, e^{-j1.05 - j\frac{4\pi}{0.08} 0.2664}$$

$$= 0.62\, e^{-j42.9} = 0.29 + j0.55$$

$$Z_{in} = Z_0 \frac{1+\Gamma(-l)}{1-\Gamma(-l)} = 50\,\frac{1+[0.29+j0.55]}{1-[0.29+j0.55]} = 38.1 + j68.0\ \Omega$$

or: Using impedance transformation equation

$$Z_{in} = Z_0 \frac{Z_L \cos\beta l + jZ_0 \sin\beta l}{Z_0 \cos\beta l + jZ_L \sin\beta l}$$

$$\beta l = \frac{2\pi}{0.08} 0.2664 = 20.923, \quad \cos\beta l = -0.482, \quad \sin\beta l = 0.876$$

$$Z_{in} = 50\,\frac{(40-j70)(-0.482) + j50\times 0.876}{50\times(-0.482) + j\underbrace{\cancel{50}}_{40-j70}\times(0.876)} = 38.1 + j68.0\ \Omega$$

The impedance transformation formula can also be written as

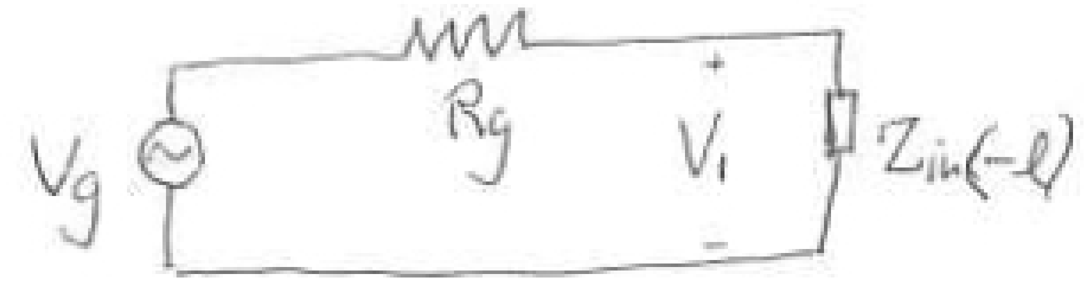
$$Z_{in}(-l) = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

3. For what min value of $l > 0$, $Z_{in}(-l) = Z_L$?

when $\tan(\beta l) = 0$, we have $Z_{in}(-l) = Z_L$

solve: $\beta l = \pi$, $l = \frac{\lambda}{2} = 4$ cm

4. Draw an equivalent circuit at $z=-l$ using $Z_{in}(-l)$ to replace the transmission line and the load. ③



5. Using the equivalent circuit, calculate the voltage at the input to the transmission line (i.e. at $z=-l$)

Solution: it is marked as V_1 in 4 (above)

$$V_1 = \frac{Z_{in}(-l)}{Z_{in}(-l)+Z_0} V_g = \frac{38.1+j68.0}{38.1+j68.0+30} 120$$

$$= 93.6+j26.4 \ (V)$$

Exercise

Repeate the above for a) $Z_L = 30+j40\ \Omega$

b) $Z_L = 60-j10\ \Omega$

Ans

	Γ_L	Φ_L	VSWR	z_{vm}	$\Gamma(-l)$	$Z_{in}(-l)$	V_1
a	$0+j0.5$	90°	3	-0.03_m	$-0.42-j0.27$	$17.9-j12.8\ \Omega$	$49.9-j18.7$ [V]
b	0.098 $-j0.082$	-39.8°	1.29	0.0156_m	$0.0165+j0.127$	$50+j12.9\ \Omega$	$76.1+j7.1$ (V)

6. Using Smith Chart to perform operations for 1. and 2.

Solution: (a) The normalized $z_L = \frac{Z_L}{Z_0} = 0.8-j1.4$

⟨See sketch on P. 4⟩, (b) place z_L on chart at A.

(c) Read ANGLE of point A: $\Rightarrow \Phi_L$

(d) Measure $|0A|$ Compare with bottom line $\Rightarrow |\Gamma|$

(e) Read VSWR

(f) WTG(z_L) = 0.334, WTG(z_{in}) = $0.334 + \frac{0.2664}{0.08} = 3.664 = 3.5 + \underline{0.164}$