Lecture-17

Kalman Filter

Main Points

- Very useful tool.
- It produces an optimal estimate of the state vector based on the noisy measurements (observations).
- For the state vector it also provides confidence (certainty) measure in terms of a covariance matrix .
- It integrates estimate of state over time.
- It is a sequential state estimator.

State-Space Model

State-transition equation

$$\mathbf{z}(k) = \Phi(k, k-1)\mathbf{z}(k-1) + \mathbf{w}(k)$$

State model error With covariance Q(k)

State Vector

Measurement (observation) equation

$$\mathbf{y}(k) = \mathbf{H}(k)\mathbf{z}(k) + \mathbf{v}(k)$$

Measurement Vector

Observation Noise with covariance R(k)

Kalman Filter Equations

State Prediction

$$\hat{\mathbf{z}}_b(k) = \Phi(k, k-1)\hat{\mathbf{z}}_a(k-1)$$

Covariance Prediction

$$\mathbf{P}_b(k) = \Phi(k, k-1)\mathbf{P}_a(k-1)\Phi^T(k, k-1) + \mathbf{Q}(k)$$

Kalman Gain

$$\mathbf{K}(k) = \mathbf{P}_b(k)\mathbf{H}^T(k)(\mathbf{H}(k)\mathbf{P}_b(k)\mathbf{H}^T(k) + \mathbf{R}(k))^{-1}$$

State-update

$$\hat{\mathbf{z}}_a(k) = \hat{\mathbf{z}}_b(k) + \mathbf{K}(k)[\mathbf{y}(k) - \mathbf{H}(k)\hat{\mathbf{z}}_b(k)]$$

Covariance-update

$$\mathbf{P}_a(k) = \mathbf{P}_b(k) - \mathbf{K}(k)\mathbf{H}(k)\mathbf{P}_b(k)$$

Two Special Cases

- Steady State

$$\Phi(k,k-1)=\Phi$$
$$\mathbf{Q}(k)=\mathbf{Q}$$
$$\mathbf{H}(k)=\mathbf{H}$$
$$\mathbf{R}(k)=\mathbf{R}$$

- Recursive least squares

$$\Phi(k,k-1)=\mathbf{I}$$
$$\mathbf{Q}(k)=0$$

Comments

- In some cases, state transition equation and the observation equation both may be non-linear.
- We need to linearize these equation using Taylor series.