

## **Today's topics:**

- **Proof that a given function is injective or surjective.**
- **Composite functions**
- **Properties of the composite functions.**
- **Inverse functions.**
- **Proofs with functions.**

Let  $A = \mathbb{R} - \{1\}$  and  $f : A \rightarrow A$  be defined by the formula:

$$f(x) = \frac{x+1}{x-1}$$

Prove that  $f$  is injective and surjective.

1) injective: Let  $x$  and  $y$  be arbitrary elements of  $A$  such that  $f(x) = f(y)$ , (1). We must show that (1) implies  $x = y$

$$\frac{x+1}{x-1} = \frac{y+1}{y-1}$$

by assumption

$$(x+1)(y-1) = (y+1)(x-1)$$

since  $(x-1) \neq 0$  and  $(y-1) \neq 0$

$$xy + y - x - 1 = yx + x - y - 1$$

by algebra

$$y - x = x - y$$

$$x = y$$

**2) surjective: for any  $y$  there exists  $x \in A$  , such that  $f(x)=y$  .**

**Take arbitrary  $y \in A$  and find  $x \in A$  from  $f(x)=y$**

$$y = \frac{x+1}{x-1}$$

$$y(x-1) = x+1$$

$$yx - y = x + 1$$

$$x(y-1) = y+1$$

$$x = \frac{y+1}{y-1} \in A$$