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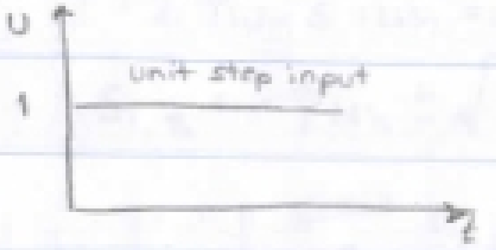
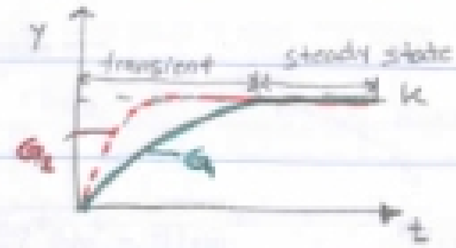
1st order

$$\frac{Y(s)}{U(s)} = G(s) = \frac{k}{\tau s + 1}$$





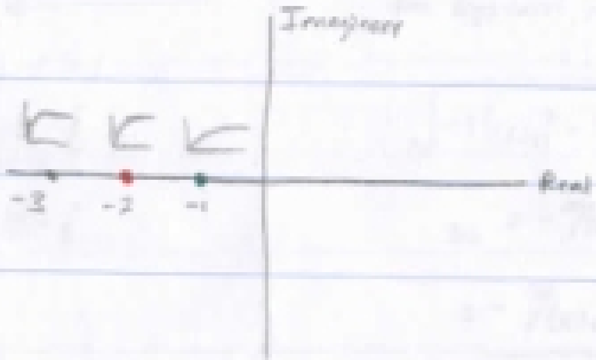
$G_1 = \frac{k}{s+1}$

$G_2 = \frac{k}{\frac{1}{2}s+1}$

*denominator of the TR defines how the graph looks in the transient region

$\tau s + 1 = 0$

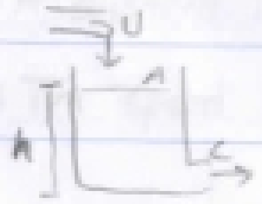
$s = \frac{-1}{\tau}$ → Poles, values of s that will make denominator of TR go to 0



for G_1; $s = -1$ → slower transient response

for G_2; $s = -2$ → faster transient response

① moving to the left on the complex plane means a faster transient response

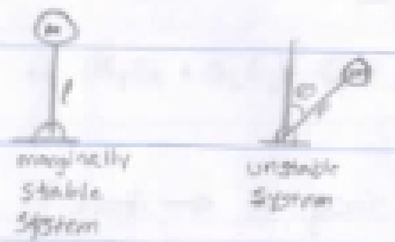


$G(s) = \frac{H(s)}{U(s)} = \frac{1}{As + C}$

Pole: $s = -\frac{C}{A}$

② Positive Real Poles mean unstable system

③ A pole at $s=0$; marginally stable



$$ax^2+bx+c=0$$
$$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

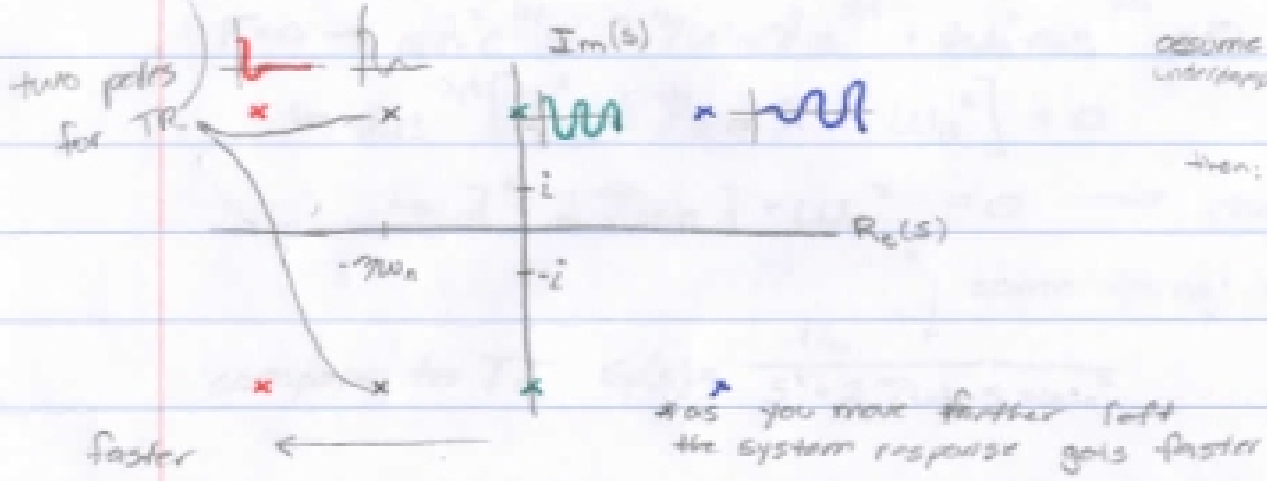
$$G(s) = \frac{\frac{1}{m}}{s^2 + 2\eta\omega_n s + \omega_n^2}$$

$$s^2 + 2\eta\omega_n s + \omega_n^2 = 0$$

$$s_{1,2} = -\eta\omega_n \pm \sqrt{\eta^2\omega_n^2 - \omega_n^2}$$ correct

check

$$s_{1,2} = \frac{-2\eta\omega_n \pm \sqrt{4\eta^2\omega_n^2 - 4\omega_n^2}}{2} = -\eta\omega_n \sqrt{\eta^2\omega_n^2}$$



assume: $0 < \eta < 1$
underdamped

then: $\eta^2\omega_n^2 < \omega_n^2$

$\eta = 0 \rightarrow$ undamped
$\eta = 1 \rightarrow$ critically damped
$\eta > 1 \rightarrow$ over damped

$\sqrt{-1} = i$

$-\sqrt{-1} = -i$

$$\sqrt{-1(\omega_n^2 - \eta^2\omega_n^2)} \rightarrow \sqrt{-1}\sqrt{\omega_n^2 - \eta^2\omega_n^2} \rightarrow i\,\omega_n\sqrt{1-\eta^2}$$

so $= -\eta\omega_n \pm \sqrt{\eta^2\omega_n^2 - \omega_n^2}$ becomes

$$= \underbrace{-\eta\omega_n}_{\text{real}} \pm \underbrace{i\,\omega_n\sqrt{1-\eta^2}}_{\text{imaginary}}$$

(4) A system with complex pair poles the system osallates

(5) The speed of the response depends on the real part

$$G(s) = \frac{H_2}{W_{in}} = \frac{C_1}{A_1A_2 s^2 + (A_1C_2 + A_2C_1)s + C_1C_2}$$

$$s_{1,2} = \frac{-(A_1C_2 + A_2C_1) \pm \sqrt{(A_1C_1 + A_2C_2)^2 + 4(A_1A_2)(C_1C_2)}}{2(A_1A_2)}$$

if $(A_1C_2 + A_2C_1)^2 < 4A_1A_2C_1C_2 \rightarrow$ you would get an imaginary number and the tank will oscillate.

if not $\rightarrow$ a pair of real & negative poles which means a decaying response.

Transient Response defined by roots of characteristic equation, poles, eigenvalues

$$s_{1,2} = -\zeta\omega_n \pm \sqrt{\zeta^2\omega_n^2 + \omega_n^2} = -\zeta\omega_n \pm i\,\omega_n\sqrt{1-\zeta^2}$$

Roots of characterisic equation

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2 x = \frac{F}{m}$$

$$x(t) = ae^{\lambda t} \rightarrow \dot{x}(t) = a\lambda e^{\lambda t} \rightarrow \ddot{x}(t) = a\lambda^2 e^{\lambda t}$$

$$F=0 \rightarrow a\lambda^2 e^{\lambda t} + 2\zeta\omega_n a\lambda e^{\lambda t} + \omega_n^2 a e^{\lambda t} = 0$$

$$\hookrightarrow ae^{\lambda t}\left[\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2\right] = 0$$

$$\hookrightarrow \lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2 = 0 \longrightarrow \text{characteristic equation}$$

same thing!

compare to TF $G(s) = \dfrac{1/m}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

State space : eigenvalues of matrix A

$$A = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \qquad |\lambda I - A| = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = ad - bc = 0$$

$$|\lambda I - A| = 0$$

$$\lambda I = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \rightarrow \lambda I - A = \begin{bmatrix} \lambda & -1 \\ \omega_n^2 & \lambda + 2\zeta\omega_n \end{bmatrix}$$

$$|\lambda I - A| = \lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2 = 0$$