Steady state – all voltages and currents are constant but this occurs some time after the circuit has settled

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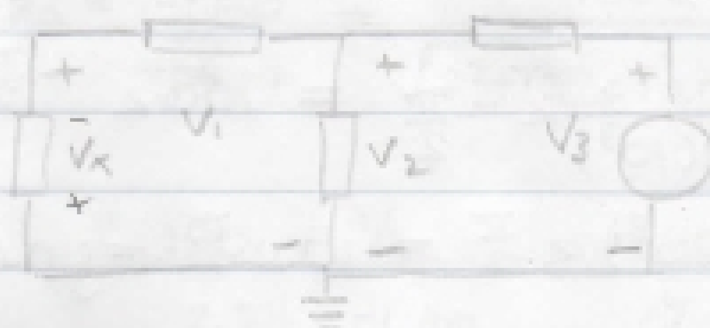
* Review of phasors — March 25, 2015

$v(t) = A\cos(\omega t + \theta) \rightarrow V(j\omega) = Ae^{j\theta} = A\cos\theta + jA\sin\theta = A\angle\theta$

Rectangular — polar

Ex.



$V_1 = 20\angle 30°$, $V_2 = 10\angle 30°$, $V_3 = 5\angle 120°$

$V_x + V_1 + V_2 = 0$

KVL:

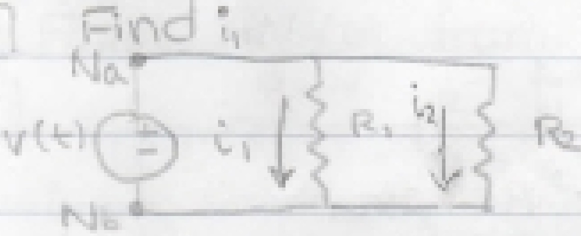
$V_x + V_1 = 0 \rightarrow V_x = V_1$

$V_x = 20\angle 150°$

Impedance (z):

- Resistor $\rightarrow R\angle 0°$

Ex. Find i_1



$R_1 = R_2 = 100\,\Omega$

$v(t) = 30\cos t(10t + \frac{\pi}{2})\ V$

$V(j\omega) = 30\angle 90°\ V$

$I_1(j\omega) = \frac{V(j\omega)}{Z(j\omega)} = \frac{30\angle 90°}{R\angle 0°} = \frac{30\angle 90°}{100\angle 0°} = 0.3\angle 90°\ A$

$i(t) = 0.3\cos t(10t + \frac{\pi}{2})\ A$

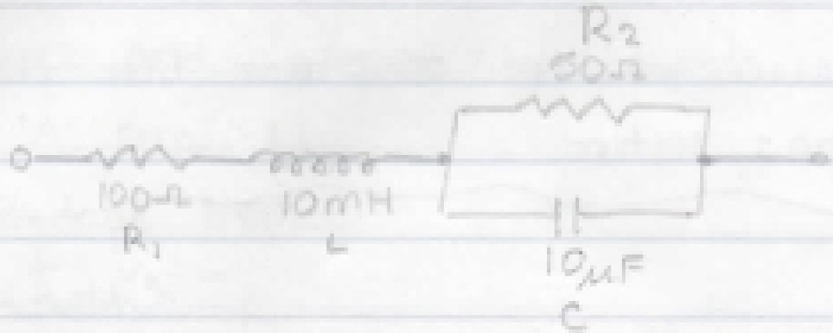
- Inductor $\rightarrow \omega L\angle 90°$
- Capacitor $\rightarrow \frac{1}{C\omega}\angle -90°$

The impedance equation: $z = \frac{V}{I} \rightarrow V = ZI$

Summary

	(Degrees) Impedance	(Radians) $Z(j\omega)$ in Ω Imp.	(Rectangular) Imp.
Resistor	$R\angle 0°$	$R\angle 0$	R
Inductor	$\omega L\angle 90°$	$\omega L\angle \pi/2$	$j\omega L$
Capacitor	$\frac{1}{C\omega}\angle -90°$	$\frac{1}{C\omega}\angle -\pi/2$	$\frac{-j}{C\omega}$

Ex.



Find impedances!

$\omega = 2\pi f = 159.15\pi = 1000$ rad/sec

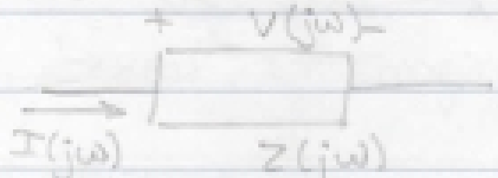
$Z_{R1} = 100\angle 0°$

$Z_{R2} = 50\angle 0°$

$Z_C = \frac{1}{\omega C}\angle -\frac{\pi}{2} = \frac{1}{(10\times10^{-6})(10^3)}\angle -\frac{\pi}{2} = 100\angle -\frac{\pi}{2} = -j100\Omega$

$Z_L = \omega L\angle \frac{\pi}{2} = (10\times10^{-3})(10^3)\angle\frac{\pi}{2} = 10\angle\frac{\pi}{2} = j10\Omega$

Ex.



Given: $V = 0.1\angle 0°$ V

Find Current:

- If $Z = 100\angle 0°$

  $I = \frac{V}{Z} = \frac{0.1\angle 0°}{100\angle 0°} = 1\angle 0°$ mA → Resistor

- If $Z = 10\angle \pi/2$

  $I = \frac{0.1\angle 0°}{10\angle \pi/2} = 10\angle -\pi/2$ mA → Inductor

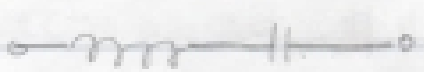
- If $Z = 100\angle -\pi/2$

  $I = \frac{0.1\angle 0°}{100\angle -\pi/2} = 1\angle \pi/2$ mA → Capacitor

* Combining Impedance

- Series → $Z_{eq} = Z_1 + Z_2$
- Parallel → $Z_{eq} = \left(\frac{1}{Z_1} + \frac{1}{Z_2}\right)^{-1}$

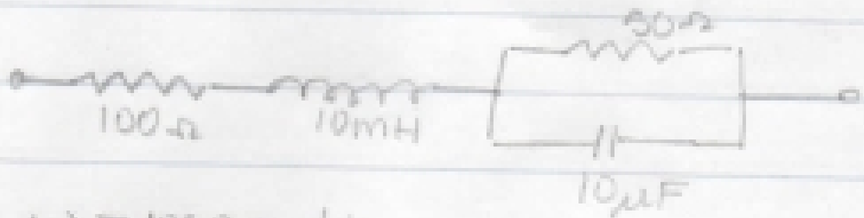
Ex.



$Z = Z_1 + Z_2$

$= \omega L\angle \pi/2 + \frac{1}{\omega C}\angle -\pi/2$

Ex.



$\omega = 1000 \text{ rad/s}$

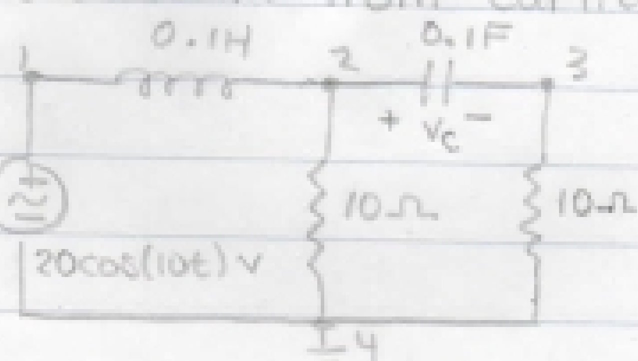
$$\frac{1}{Z_a} = \frac{1}{50} + \frac{1}{-j100} = \frac{-j2+1}{-j100}$$

$$Z_a = \frac{-j100}{-j2+1} = \frac{100\angle -\pi/2}{\sqrt{2^2+1^2}\ \angle \tan^{-1}(-2)}$$

$$= \frac{100\angle -\pi/2}{2.24\angle -1.107} = \boxed{40 - j20\,\Omega}$$

* Node Voltage Method
  - Transform phasors + impedance
  - Follow NVM from earlier in semester

Ex.



$j\omega L = j1\,\Omega$

$-j/\omega C = -\frac{j}{1} = -j\,\Omega^{-1}$

$V_s = 20\angle 0°\ V$

$\omega = 10 \text{ rad/s}$

$m=1,\ n=4$

Out of node 2 : $I_n = \frac{V_2 - V_1}{j1\Omega} + \frac{V_2}{10\Omega} + \frac{V_2 - V_3}{-j\Omega} = 0$

" " " 1 : $I_n = \frac{V_3 - V_2}{-j\Omega} + \frac{V_3}{10\Omega} = 0$