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y_1, y_2

$y'' + P(\tau)y' + Q(\tau)y = 0$

$W(y_1, y_2) = Ce^{\int -P(\tau)d\tau}$

$W(y_1, y_2) = y_1y_2' - y_1'y_2$

$y'' + 2y' + y = 0$

$r^2 + 2r + 1 = 0$

$r = -1$

$y_1 = e^{-\tau}$

assume $y_2 = f(\tau)y_1 = fe^{-\tau} = y$

$y' = f'e^{-\tau} - fe^{-\tau}$

$y'' = (f''e^{-\tau} - f'e^{-\tau}) - (f'e^{-\tau} - fe^{-\tau}) = f''e^{-\tau} - 2f'e^{-\tau} + fe^{-\tau}$

$f''e^{-\tau} - 2f'e^{-\tau} + fe^{-\tau} + 2(f'e^{-\tau} - fe^{-\tau}) + fe^{-\tau} = 0$

$f''e^{-\tau} = 0$

$f'' = 0$

$f' = C$

$f = C_1\tau + C_2$

$y_1 = e^{-\tau}$

$y_2 = (C_1\tau + C_2)e^{-\tau}$

$y = K_1e^{-\tau} + C_1\tau e^{-\tau} + C_2e^{-\tau}$

$\boxed{y = C_2e^{-\tau} + C_1\tau e^{-\tau}}$

$y'' - 8y' + 16y = 0$

$r = 4$

$y_1 = e^{4\tau}$

$y_2 = C\tau e^{4\tau}$

repeated root

$\boxed{y = C_1e^{r\tau} + C_2\tau e^{r\tau}}$

$y'' + 2y' + y = 0$

$r^2 + 2r + 1 = 0$

$r = -1$

$y_1 = e^{-\tau}$

$W = e^{\int -2d\tau}$

$W = e^{-2\tau}$

$W = y_1y_2' - y_1'y_2$

$y_1y_2' - y_1'y_2 = e^{-2\tau} \longrightarrow e^{-\tau}y_2' + e^{-\tau}y_2 = e^{-2\tau}$

$y_2' + y_2 = e^{-\tau}$ $\quad$ $\mu(\tau) = e^{\int 1\,d\tau} = e^{\tau}$

$e^{\tau}y_2' + e^{\tau}y_2 = 1$

$(e^{\tau}y_2)' = f'$

$e^{\tau}y_2 = \tau + C$

$y = \tau e^{-\tau} + Ce^{-\tau}$

$y = C_2e^{-\tau} + C_3\tau e^{-\tau} + C_4e^{-\tau}$

$ay'' + by' + cy = 0$

$r_1 \neq r_2$ and not complex

$y = C_1e^{r_1\tau} + C_2e^{r_2\tau}$

$r_1 \neq r_2$ and complex $\quad r_1 = a + bi \Leftarrow e^{r_1x} = e^{(a+bi)x} = e^{ax}e^{bxi}$

$y = e^{ax}(C_1\cos(bx) + C_2\sin(bx))$

$y'' + y' + 3y = 0$

$r^2 + r + 3 = 0$

$r = \dfrac{-1 \pm \sqrt{1^2 - 4(1)(3)}}{2}$

$= \dfrac{-1 \pm \sqrt{11}\,i}{2}$

$e^{-\frac{1}{2}x}\left(\cos\frac{\sqrt{11}}{2}x + C_2\sin\frac{\sqrt{11}}{2}x\right)$

$x^2y'' + 3xy' - 3y = 0$ = $y'' + \frac{3}{x^2}y - \frac{3}{x^2}y = 0$

$y_1 = \frac{1}{x}$ Find y_2

$W = y_1y_2' - y_1'y_2 = Ce^{\int -P(x)dx}$

$Ce^{-\int \frac{3}{x^2}dx}$

$Ce^{-3\int \frac{1}{x}dx} = \frac{1}{x}y_2' + \frac{1}{x^2}y_2$

$e^{-3\ln x} = \frac{1}{x}y_2' + \frac{1}{x^2}y_2$

$x^{-3} = \frac{1}{x}xy_2' + \frac{1}{x^2}y_2$

$y_2' + \frac{1}{x}y_2 = x^{-1/2}$

$M(x) = e^{\int \frac{1}{x}dx} = e^{\ln x} = x$

$xy_2' + y_2 = x^{1/2}$

$(xy_2)' = x^{1/2}$

$xy_2 = \frac{2}{3}x^{3/2}$

$y_2 = \frac{2}{3}x^{1/2}$

$y = C_1\left(\frac{1}{x}\right) + C_2x^{1/2}$