PRIMER on ERRORS & Analysis – part II

8.13 U.Becker Sept.,2006

Recall:

- ♦ Accuracy = closeness to "truth" (NIST)
- ♦ Precision = quality of data ~ 1/√N usually

- ♦ systematic error = reproducible, but deviation in system, apparatus, environment or calibration
- ♦ Statistical error = due to finite statistics ~ 1/√N

Mean:

$$\langle x \rangle = \frac{1}{N}\sum_{i=1}^{N} x_i$$

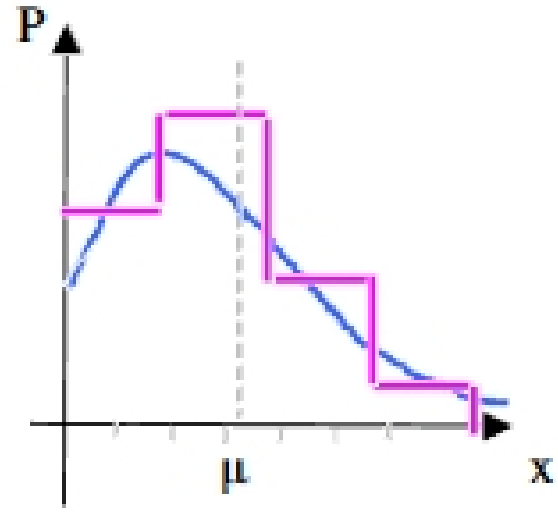
Variance:

$$s_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \langle x \rangle)^2$$

For independent measurements

nothing depends on the previous operations or history.

$$P(x,\mu)=\frac{\mu^x}{x!}e^{-\mu}=\frac{(t/\tau)^r}{r!}e^{-t/\tau}$$

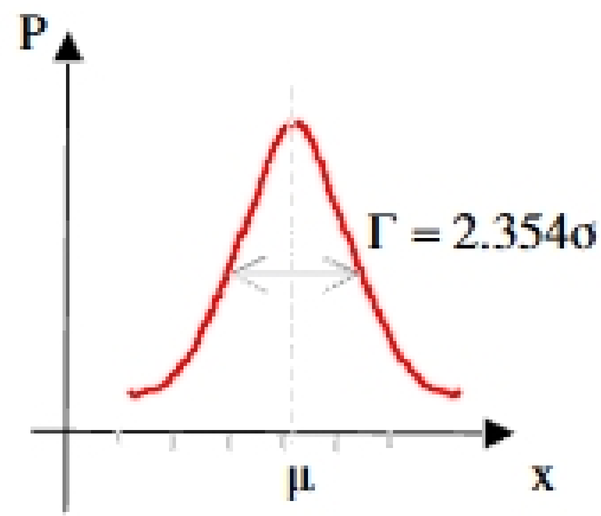


Poisson Distr.

mean $= \mu$
variance $\sqrt{\mu}$

The probability dP to observe no count in dt for random events spaced τ on average is: dP(0;t,τ)=-P(0;t,τ) dt/τ. The obvious solution is: $P(0;t,\tau) = e^{-t/\tau}$

Which means short intervals are more likely!!



$$P(x;\mu,\sigma)=\frac{1}{2\pi\sigma}e^{\frac{(x-\mu)^2}{2\sigma^2}}$$

Gaussian

mean $= \mu$
variance $= \sigma = \sqrt{\mu}$
standard deviation

Applications are numerous, folding two Gaussians yields a Gaussian again. The errors simply add in quadrature.

PROPAGATION OF ERRORS   Bevington ch.3

You measure $u \pm \sigma_u$ and $v \pm \sigma_v$
but evaluate $x = au + bv$ or $x = au/v$ and want the error for those

Taylor expansion: $x_i - \langle x \rangle = (u_i - \langle u \rangle)\frac{\partial x}{\partial u} + (v_i - \langle v \rangle)\frac{\partial x}{\partial v} + \ldots$

then
$$\sigma_x^2 = \lim_{N \to \infty} \frac{1}{N} \sum \left[(u_i - \langle u \rangle)\frac{\partial x}{\partial u} + (v_i - \langle v \rangle)\frac{\partial x}{\partial v} \right]^2$$

$$= \sigma_u^2 \left(\frac{\partial x}{\partial u} \right)^2 + \sigma_v^2 \left(\frac{\partial x}{\partial v} \right)^2 + 2\sigma_{uv}^2 \left(\frac{\partial x}{\partial u} \right)\left(\frac{\partial x}{\partial v} \right)$$

= 0 if uncorrelated,otherwise σ_{uv} covariance matrix -difficult

Sum&Diff: $x = au \pm bv$ -> $\sigma_x^2 = a^2\sigma_u^2 + b^2\sigma_v^2 \pm 2ab\sigma_{uv}^2$

8.13

Prod.&Div: $x = auv$ or $a\,u/v$ -> $\frac{\sigma_x^2}{x^2} = \frac{\sigma_u^2}{u^2} + \frac{\sigma_v^2}{v^2} + \frac{2\sigma_{uv}^2}{uv}$

Powers: $x = a\,T^4$ -> $\frac{\sigma_x}{x} = \pm \boxed{4} \frac{\sigma_T}{T}$ watch out!