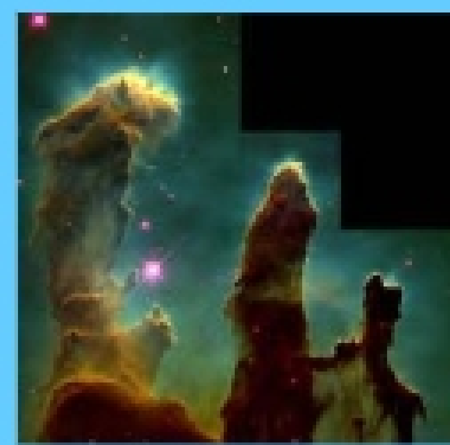
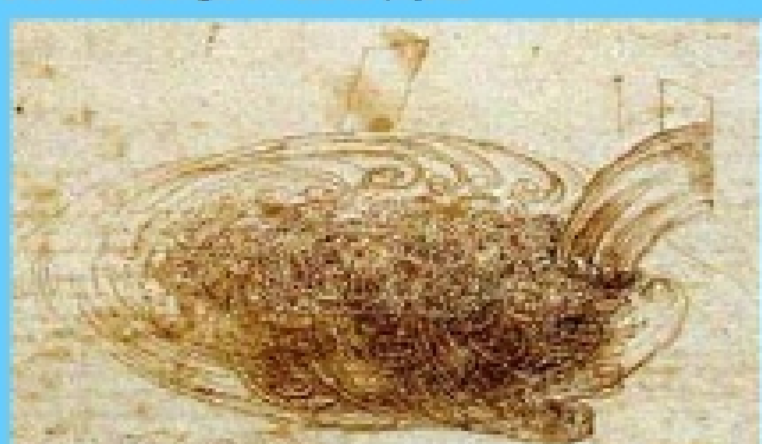

*"Perhaps the fundamental equation that describes the swirling nebulae and the condensing, revolving, and exploding stars is just a simple equation for the hydrodynamic behavior of nearly pure hydrogen gas"*— Richard Feynman, Nobel Laureate

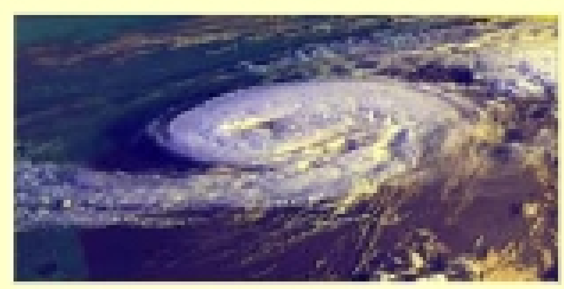


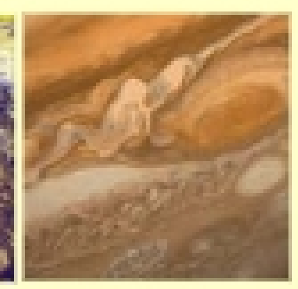
Turbulence is widespread, indeed almost the rule, in the flow of fluids. It is a complex phenomenon, for which the development of a satisfactory theoretical framework has been one of the greatest unsolved challenges of classical physics.



The first scientific investigations of fluid turbulence are generally attributed to Leonardo Da Vinci.

Turbulence exists in a wide range of contexts such as the motion of submarines, ships and aircraft, pollutant dispersion in the earth's atmosphere and oceans, heat and mass transport in engineering applications as well as geophysics and astrophysics [See, e.g., D.J. Tritton, *Physical Fluid Dynamics*. Clarendon Press, Oxford (1988)].





The problem is also a paradigm for strongly nonlinear systems, distinguished by strong fluctuations and strong coupling among a large number of degrees of freedom. [G. Falkovich, K.R. Sreenivasan, *Phy. Today* 59, 43 (2006)].

Turbulence is particularly useful because the equations of motion are known exactly and can be simulated with precision. And so, even distant areas--perhaps even market fluctuations [B.B. Mandelbrot, *Scientific American* 280, 50 (1999)]—may benefit from a better understanding of it.

The complexity of the underlying equations, however, has precluded much analytical progress, and the demands of computing power are such that routine simulations of large turbulent flows has not yet been possible. Thus, the progress in the field has depended more on experimental input. This experimental input in turn points in part to a search for optimal test fluids, and the development and utilization of novel instrumentation. These aspects will be discussed in this lecture.

While it is not possible to predict turbulent motion in all details as a function of *both time and position*, turbulence can be well-characterized *statistically*, with reproducible *average* values of certain quantities.

Let's take a quick glance at Newton's second law for hydrodynamics, including turbulence (Navier-Stokes)

$$\frac{\partial v}{\partial t} + (v \cdot \nabla)v = -\frac{1}{\rho}\nabla p + \iota\nabla^2 v$$

$$\frac{\partial v'}{\partial t} + (v' \cdot \nabla')v' = -\nabla' p' + \frac{1}{Re}\nabla'^2 v'$$

*Re = UL/ν*

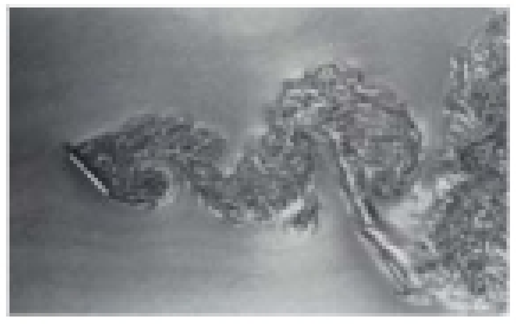
$$v' = \frac{v}{U} \quad r' = \frac{r}{L} \quad t' = \frac{tU}{L}? \quad p' = \frac{p}{\rho U^2}?$$

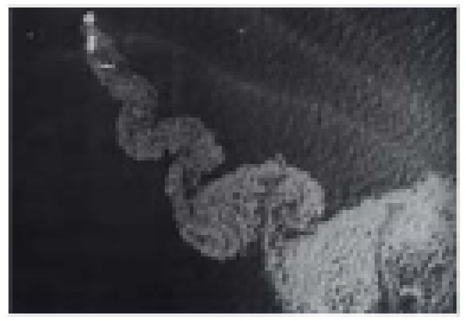
a b

Flow past a circular cylinder. (a) Re = 26. (b) Re = 2000.

**The Reynolds number is a manifestation of dynamical similarity.** That is, if we consider two simple flows that are geometrically similar, then they are dynamically identical if the corresponding *Re* is the same for both, *regardless of the specific velocities, lengths and fluid viscosities involved.*

Matching such parameters between laboratory testing of a model and the actual full-scale object (the prototype) is the principle upon which aerodynamic model-testing is based.





Above left: wake behind a flat plate inclined 45 degrees to the direction of the flow (left to right). Above right: A foundered ship in the sea inclined 45 degrees to the direction of the current.

Q. Two flows are dynamically similar if---

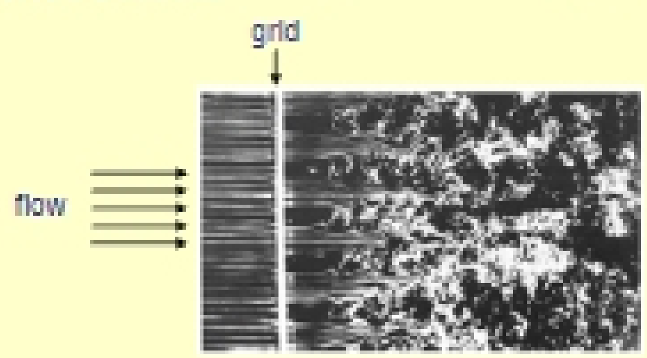
a) the fluids have the same kinematic viscosity

b) the flows have the same Re

c) the flows have large Re>>1

## Making Turbulence in the laboratory

Traditionally, large arrays of wake-producing "cylinders"-- such as we saw above-- are placed in the flow in order to generate something approaching homogeneous and isotropic turbulence.

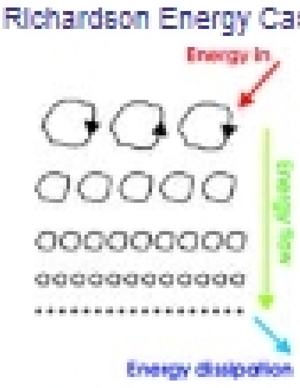



Initial turbulent forcing at scale of the mesh size M.

Here, flow at speed *U* through a grid of crossed tines with mesh size *M*, generates a turbulent wake, injecting kinetic energy initially on that scale.

## Where does the injected energy go?

Richardson Energy Cascade




Eddies produced by flow (e.g., through a grid at length scale M= mesh size)

↓

Cascade of energy in inertial range: **local transfer of energy** (due to nonlinear inertial term in NS eqn.).

↓

Viscous dissipation at small scales for which local Re~1.

For "inertial range" between large energy injection scale L and the smallest dissipation scale $\eta$ the energy spectrum (in k-space) is $E(k) = C\,\varepsilon^{2/3} k^{-5/3}$ where $\varepsilon$ is the rate of energy transfer per unit mass. (E(k) is the energy contained in the wavenumber shell between k and k+dk.

Smallest scale given by $\eta \sim L Re^{-3/4}$.

If there are any universal statistical properties of turbulence, it is reasonable to look for them as Re→∞, since the separation between the energy-injection scales and the dissipative scales increases with Re.

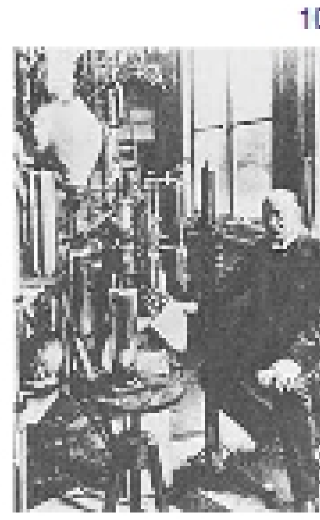
Q. As the viscosity gets smaller

a) The dissipation length scale grows

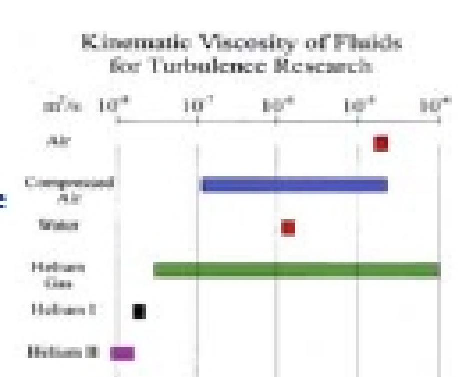
b) The dissipation length remains the same

c) The dissipation length becomes smaller

## 100 years ago...



**Leiden, 1908**: Kamerlingh Onnes succeeds in liquifying helium, an element first identified spectroscopically in India in 1868 during a total eclipse of the sun. This leads to the discovery of superconductivity a few years later, superfluidity a few decades later, and many diverse applications in science and engineering, including the experimental study of fluid turbulence:




$$Re = \frac{UL}{\nu}$$

*Arbitrarily increasing U* can introduce Mach number (=U/c).

Increasing *L* has its obvious limits.

Helium has the lowest kinematic viscosity of any fluid.

### Alternative approaches

It is reasonable to ask why nature's "laboratories", such as the atmosphere and oceans, cannot be instrumented and studied for turbulence dynamics. They can, but this is not a substitute for controlled laboratory studies when questions become sharp and a deeper understanding is required.

Direct numerical simulations (DNS) can be performed in which the appropriate equations are solved on a computer without making any approximation. The range of scales needing to be well resolved, however, grows as $Re^{3/4}$, and thus $Re^{9/4}$ in 3 dimensions, severely hampering DNS efforts. The state of the art in DNS is about $Re \sim 10^4$, or about 3-4 orders of magnitude lower than the Re corresponding to a typically commercial jet aircraft, and the same amount for most atmospheric and oceanic flows.

Under certain circumstances, large eddy simulations (LES)--which compute only the large scales but model the small scales-- do better in terms of providing useful information, but they are not satisfactory as universal recipes. The state of the art in computer hardware is years away from allowing us to address the most important problems in natural and engineering fluid turbulence.

### *Continuum Approximation*

An interesting question that has been raised at various times is whether the increasingly small scales that develop as the Reynolds number is increased render tenuous the continuum approximation of classical hydrodynamics; i.e., does one have to worry about aspects of molecular motion? U. Frisch has addressed this question in a general context and demonstrated that the ratio of the dissipation scale η to the molecular mean free path *grows* with increasing Reynolds number. The hydrodynamic approximation thus should become better applicable at higher *Re*. However, there are no definitive experiments yet to confirm this.

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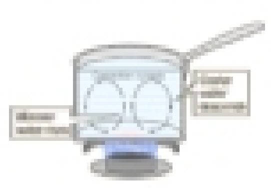
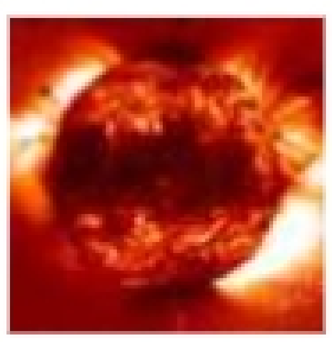
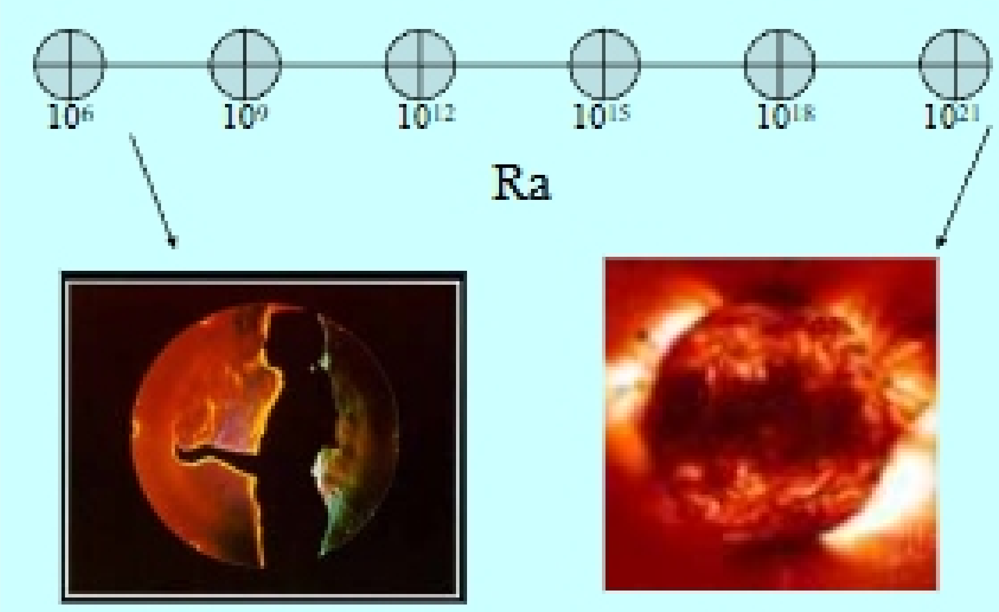
### Thermally-generated turbulence

Like other flows, convection is often turbulent, and in fact, turbulent thermal convection plays a prominent role in the energy transport within stars, atmospheric and oceanic circulations, the generation of the earth's magnetic field, and also innumerable engineering processes in which heat transport is an important factor. *It is possibly the most ubiquitous fluid flow in the universe.*

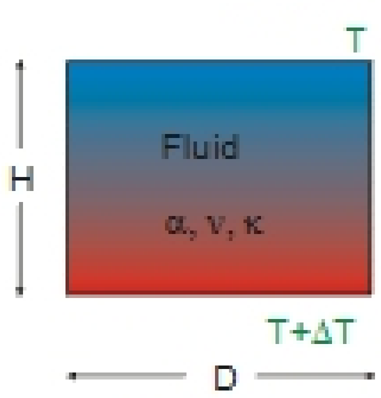
*Thermal convection transports and mixes heat from the bottom of a cooking pot to the top.*








### Rayleigh-Benard Convection




α fluid thermal expansion coefficient

ν fluid kinematic viscosity

κ fluid thermal diffusivity

The control parameters for convection:

$$Ra = \frac{g\alpha\Delta T H^3}{\nu\kappa} \qquad Pr = \frac{\nu}{\kappa} \qquad \Gamma = \frac{D}{H}$$

Rayleigh number — Prandtl number — Aspect ratio

RBC near onset: linear temperature gradients, up/down symmetry, steady patterns