

Chapter 6: Practice/review problems

The collection of problems listed below contains questions taken from previous MA123 exams.

Extreme values problems on a closed interval

[1]. Suppose $f(t) = \begin{cases} \sqrt{4-t} & \text{if } t < 4 \\ \sqrt{t-4} & \text{if } t \geq 4. \end{cases}$

Find the minimum of $f(t)$ on the interval $[0, 6]$.

- (a) 0 (b) 2 (c) 4 (d) 6 (e) 8

[2]. Let $g(s) = \frac{s-1}{s+1}$. Find the maximum of $g(s)$ on the interval $[0, 2]$.

- (a) $-1/3$ (b) 0 (c) $1/3$ (d) $2/3$
 (e) Neither the maximum nor the minimum exists on the given interval.

[3]. Suppose $f(t) = \begin{cases} t^2 - 2t + 2 & \text{if } t < 1 \\ t^3 & \text{if } t \geq 1. \end{cases}$

Find the minimum of $f(t)$ on the interval $[0, 2]$.

- (a) -1 (b) 0 (c) 1 (d) 2 (e) 8

[4]. Let $f(x) = 3x^2 + 6x + 4$. Find the maximum value of $f(x)$ on the interval $[-2, 1]$.

- (a) 5 (b) 7 (c) 9 (d) 13 (e) -1

[5]. Let $G(x) = \begin{cases} (x-3) + 6 & \text{if } x \geq 3 \\ -(x-3) + 6 & \text{if } x < 3. \end{cases}$

Find the minimum of $G(x)$ on the interval $[-10, 10]$.

- (a) 3 (b) 1 (c) -6 (d) 19 (e) 6

[6]. Let $g(s) = \frac{1}{s+1}$. Find the maximum of $g(s)$ on the interval $[0, 2]$.

- (a) -1 (b) 0 (c) 1 (d) 2
 (e) Neither the maximum nor the minimum exists on the given interval.

[7]. Find the minimum value of $f(x) = x^3 - 3x + 3$ on the interval $[-2, 4]$.

- (a) 2 (b) 1 (c) 0 (d) -1 (e) -2

- [8]. Find the maximum of $g(t) = |t + 4| + 10$ on the interval $[-12, 12]$.
- (a) 19 (b) 20 (c) 24 (d) 26 (e) 28
- [9]. Find the minimum value of $f(x) = \sqrt{x^2 - 2x + 16}$ on the interval $[0, 5]$.
- (a) 1 (b) 2 (c) $\sqrt{15}$ (d) $\sqrt{12}$ (e) 0
- [10]. Let $f(x) = |x^2 - 1| + 2$. Find the minimum of $f(x)$ on the interval $[-3, 3]$.
- (a) 3 (b) 0 (c) 1 (d) 2 (e) -1
- [11]. Suppose $f(t) = 2t^3 - 9t^2 + 12t + 31$. Find the value of t in the interval $[0, 3]$ where $f(t)$ takes on its minimum.
- (a) 0 (b) 1 (c) 2 (d) 3
- (e) Neither the maximum nor the minimum exists on the given interval.
- [12]. Let $Q(t) = t^2$. Find a value A such that the average rate of change of $Q(t)$ from 1 to A equals the instantaneous rate of change of $Q(t)$ at $t = 2A$
- (a) 1 (b) $\frac{1}{3}$ (c) $\frac{1}{4}$ (d) $\frac{1}{5}$ (e) Does not exist

Mean Value Theorem problems

- [13]. Find the value of A such that the average rate of change of the function $g(s) = s^3$ on the interval $[0, A]$ is equal to the instantaneous rate of change of the function at $s = 1$.
- (a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\sqrt{5}$ (d) $\sqrt{6}$ (e) $\sqrt{12}$
- [14]. Suppose $k(s) = s^2 + 3s + 1$. Find a value c in the interval $[1, 3]$ such that $k'(c)$ equals the average rate of change of $k(s)$ on the interval $[1, 3]$.
- (a) -1 (b) 0 (c) 1 (d) 2 (e) 3
- [15]. Let $k(x) = x^3 + 2x$. Find a value of c between 1 and 3 such that the average rate of change of $k(x)$ from $x = 1$ to $x = 3$ is equal to the instantaneous rate of change of $k(x)$ at $x = c$.
- (a) 30 (b) 15 (c) $\sqrt{\frac{28}{3}}$ (d) $\sqrt{\frac{13}{3}}$ (e) 60

Increasing/decreasing problems

- [16]. Which function is always increasing on $(0, 2)$
- (a) $\sqrt{x} + x^2$ (b) $x + (1/x)$ (c) $x^3 - 3x$
- (d) $7 - |x|$ (e) $(x - 1)^4$

- [17]. Suppose that a function $f(x)$ has derivative $f'(x) = x^2 + 1$. Which of the following statements is true about the graph of $y = f(x)$?
- (a) The function is increasing on $(-\infty, \infty)$
 (b) The function is decreasing on $(-\infty, \infty)$
 (c) The function is increasing on $(-\infty, -1)$ and $(1, \infty)$, and the function is decreasing on $(-1, 1)$.
 (d) The function is increasing on $(-\infty, 0)$, and the function is decreasing on $(0, \infty)$.
 (e) The function is decreasing on $(-\infty, 0)$, and the function is increasing on $(0, \infty)$.
- [18]. Find the largest value of A such that the function $g(s) = s^3 - 3s^2 - 24s + 1$ is increasing on the interval $(-5, A)$.
- (a) -4 (b) -2 (c) 0 (d) 2 (e) 4
- [19]. Let $f(x) = e^{-x^2}$. Find the intervals where $f(x)$ is decreasing.
- (a) $(-\infty, 0)$ (b) $(0, \infty)$ (c) $(-\infty, -1)$
 (d) $(1, \infty)$ (e) $(-1, 1)$
- [20]. Let $f(x) = x \ln x$. Find the intervals where $f(x)$ is increasing.
- (a) $(0, \infty)$ (b) $(1, \infty)$ (c) (e, ∞)
 (d) $(1/e, \infty)$ (e) $(1/e, e)$
- [21]. Suppose the cost, $C(q)$, of stocking a quantity q of a product equals $C(q) = \frac{100}{q} + q$. The rate of change of the cost with respect to q is called the marginal cost. When is the marginal cost positive?
- (a) $q > 10$ (b) $q > 15$ (c) $q < 20$ (d) $q < 25$ (e) $q = 30$
- [22]. For which values of t is the function $t^3 - 2t + 1$ increasing?
- (a) $t > \sqrt{2/3}$ or $t < -\sqrt{2/3}$ (b) $-\sqrt{2/3} < t < \sqrt{2/3}$ (c) $0 < t < \sqrt{4/3}$
 (d) $-\sqrt{4/3} < t < 0$ (e) Never
- [23]. Suppose that $g'(x) = x^2 - x - 6$. Find the interval(s) where $g(x)$ is increasing.
- (a) $(-1, 2)$ (b) $(-\infty, -2)$ and $(3, \infty)$ (c) $(-\infty, -1)$ and $(2, \infty)$
 (d) $(-2, 3)$ (e) It cannot be determined from the information given
- [24]. Let $f(x) = xe^{2x}$. Then f is decreasing on the following interval.
- (a) $(-\infty, -1/2)$ (b) $(-1/2, \infty)$ (c) $(-\infty, 1/2)$
 (d) $(1/2, \infty)$ (e) $(-\infty, 0)$