6.118 For the piping system of Fig. P6.118, all pipes are concrete with a roughness of 0.04 inch. Neglecting minor losses, compute the overall pressure drop p1 − p2 in lbf/in^2. The flow rate is 20 ft^3/s of water at 20°C.

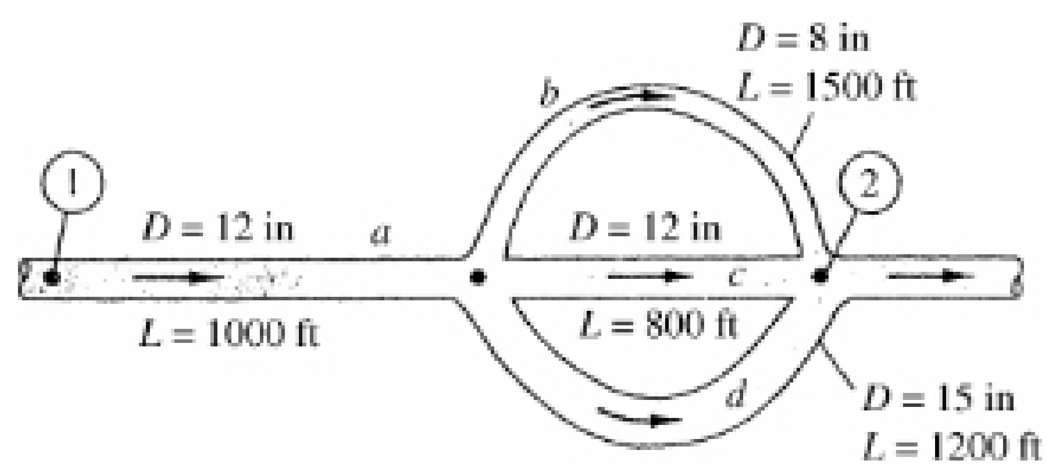

Fig. P6.118

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E−5 slug/ft·s. Since the pipes are all different make a little table of their respective L/d and ε/d:

(a)	L = 1000 ft,	d = 12 in,	L/d = *1000*,	ε/d = **0.00333**
(b)	1500 ft	8 in	*2250*	**0.00500**
(c)	800 ft	12 in	*800*	**0.00333**
(d)	1200 ft	15 in	*960*	**0.00267**

With the flow rate known, we can find everything in pipe (a):

$$V_a = \frac{Q_a}{A_a} = \frac{20}{(\pi/4)(1\text{ ft})^2} = 25.5\ \frac{\text{ft}}{\text{s}},\quad Re_a = \frac{1.94(25.5)(1)}{2.09E{-}5} = 2.36E6,\quad f_a \approx \mathit{0.0270}$$

Then pipes (b,c,d) are in parallel, each having the same head loss and with flow rates which must add up to the total of 20 ft^3/s:

$$h_{fb} = \frac{8f_bL_bQ_b^2}{\pi^2 g\, d_b^5} = h_{fc} = \frac{8f_cL_cQ_c^2}{\pi^2 g\, d_c^5} = h_{fd} = \frac{8f_dL_dQ_d^2}{\pi^2 g\, d_d^5},\quad \text{and}\quad Q_b + Q_c + Q_d = 20\ \frac{\text{ft}^3}{\text{s}}$$

Introduce L_b, d_b, etc. to find that $Q_c = 3.77Q_b(f_b/f_c)^{1/2}$ and $Q_d = 5.38Q_b(f_b/f_d)^{1/2}$

Then the flow rates are iterated from the relation

$$\Sigma Q = 20\ \frac{\text{ft}^3}{\text{s}} = Q_b[1 + 3.77(f_b/f_c)^{1/2} + 5.38(f_b/f_d)^{1/2}]$$

First guess: $f_b = f_c = f_d$: $Q_b \approx 1.97$ ft^3/s; $Q_c \approx 7.43$ ft^3/s; $Q_d \approx 10.6$ ft^3/s

Improve by computing $Re_b \approx 349000$, $f_b \approx 0.0306$, $Re_c \approx 878000$, $f_c \approx 0.0271$, $Re_d \approx 1002000$, $f_d \approx 0.0255$. Repeat to find $Q_b \approx 1.835$ ft^3/s, $Q_c \approx 7.351$ ft^3/s, $Q_d \approx 10.814$ ft^3/s. Repeat once more and quit: $Q_b \approx 1.833$ ft^3/s, $Q_c \approx 7.349$ ft^3/s, $Q_d \approx 10.819$ ft^3/s, from which $V_b \approx 5.25$ ft/s, $V_c \approx 9.36$ ft/s, $V_d \approx 8.82$ ft/s. The pressure drop is

$$p_1 - p_2 = \Delta p_a + \Delta p_b = f_a\frac{L_a}{d_a}\frac{\rho V_a^2}{2} + f_b\frac{L_b}{d_b}\frac{\rho V_b^2}{2}$$

$$= 17000 + 1800 \approx 18800 \text{ psf} \approx \mathbf{131\ \frac{lbf}{in^2}}\quad \textit{Ans.}$$

6.130 In Fig. P6.130 lengths *AB* and *BD* are 2000 and 1500 ft, respectively. The friction factor is 0.022 everywhere, and p_A = 90 lbf/in^2 gage. All pipes have a diameter of 6 in. For water at 20°C, determine the flow rate in all pipes and the pressures at points *B*, *C*, and *D*.

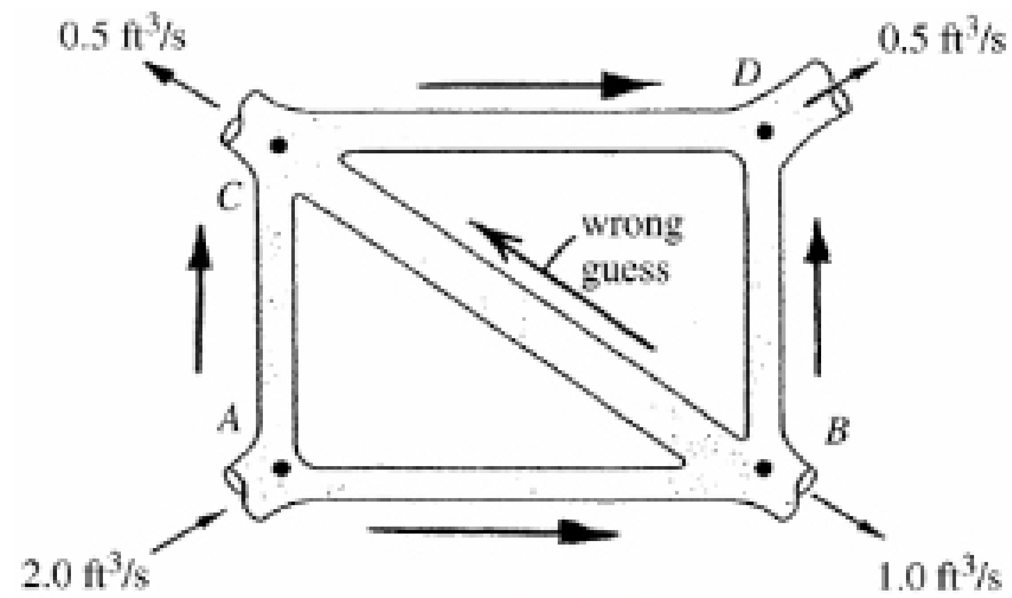

Fig. P6.130

Solution: For water at 20°C, take ρ = 1.94 slug/ft^3 and μ = 2.09E−5 slug/ft·s. Each pipe has a head loss which is known except for the square of the flow rate:

$$\text{Pipe AC:}\quad h_f = \frac{8fLQ^2}{\pi^2 g d^5} = \frac{8(0.022)(1500)Q_{AC}^2}{\pi^2(32.2)(6/12)^5} = K_{AC}Q_{AC}^2,\quad \text{where } K_{AC} \approx \mathbf{26.58}$$

Similarly, $K_{AB} = K_{CD} = 35.44$, $K_{BD} = 26.58$, and $K_{BC} = 44.30$.

$$\text{Loop ABC:}\quad 35.44Q_{AB}^2 + 44.3Q_{BC}^2 - 26.58Q_{AC}^2 = 0$$

$$\text{Loop BCD:}\quad 44.3Q_{BC}^2 + 35.44Q_{CD}^2 - 26.58Q_{BD}^2 = 0$$

$$\text{Junctions A,B,C:}\quad Q_{AB} + Q_{AC} = 2.0;$$

$$Q_{AB} = Q_{BC} + Q_{BD} + 1.0;\quad Q_{AC} + Q_{BC} = Q_{CD} + 0.5$$

After solving the above equations:

$$Q_{AB} = \mathbf{0.949\ ft^3/s}\ \text{(toward B)};\quad Q_{AC} = \mathbf{1.051\ ft^3/s}\ \text{(toward C)}$$

$$Q_{BC} = \mathbf{0.239}\ \text{(toward B)};\quad Q_{CD} = \mathbf{0.312}\ \text{(toward D)};\quad Q_{BD} = \mathbf{0.188}\ \text{(to D)}\quad \textit{Ans. (a)}$$

The pressures start at A, from which we subtract the friction losses in each pipe:

$$p_B = p_A - \rho g K_{AB}Q_{AB}^2 = 90\times144 - 62.4(35.44)(0.949)^2 = 10969\ \text{psf} \div 144 = \mathbf{76\ psi}$$

Similarly, we obtain $p_C = 11127\ \text{psf} = \mathbf{77\ psi}$; $p_D = 10911\ \text{psf} \approx \mathbf{76\ psi}$ *Ans.* (b)