Nuclear decay

Radioactivity

Nuclear decay

- Radioactivity
- Curie, Becquerel
- N radioactive nuclei
- dN decay in a time dt

$$dN \propto N$$

$$\frac{dN}{dt} = -\lambda N$$

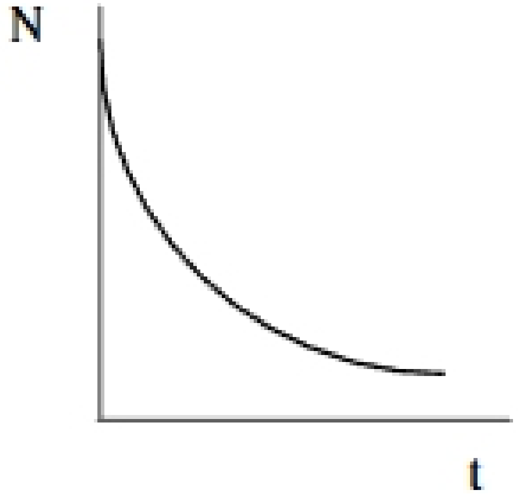
$$N(t) = N_0 e^{-\lambda t}$$

λ = decay constant = probability of a nucleus decaying per second

Half-life = time for half the nuclei to decay

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

Lifetime (average) $$\tau = \frac{1}{\lambda}$$



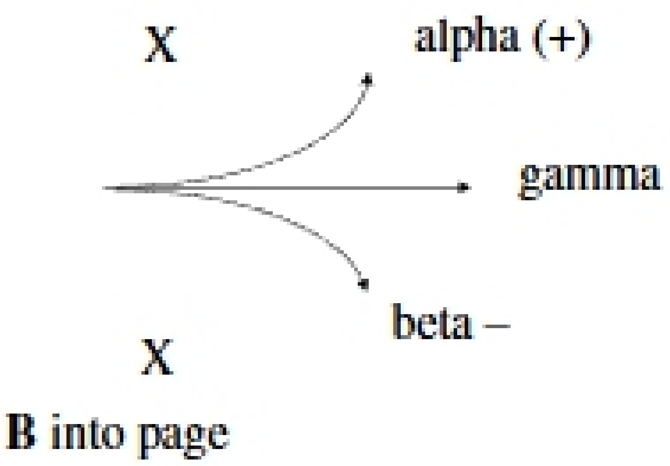
- Units: 1 Becquerel (Bq) = 1 decay/second
- 1 Curie (Ci) = 3.7×10^{10} decays/sec (1g of radium)
- 3 types of radiation emitted spontaneously

  α = ^{4}He nucleus (2 p and 2 n)

  β = e^- or e^+

  γ = high-energy photons (keV, MeV)

Effect of a magnetic field



- Decay rate ("Activity")

$$R = \left|\frac{dN}{dt}\right| = \lambda N_0 e^{-\lambda t} = \lambda N(t)$$

- Half-life e.g.

$$N_o = 20 \quad \& \quad t_{1/2} = 1\,hr$$

t (hr)	N
0	20
1	10
2	5
etc	

α decay

α decay

α-particle is a ^{4}He nucleus (2p & 2n)

- Z ↓ by 2
- N ↓ by 2
- A ↓ by 4

$${}^{A}_{Z}X_{N} \rightarrow {}^{A-4}_{Z-2}Y_{N-2} + \alpha$$

"parent" "daughter"

$${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + \alpha$$

$$t_{1/2} = 4.47 \times 10^{9}\ yrs$$

Disintegration energy

$$Q = \left(M_P - M_D - m_\alpha\right)c^2$$

$$\begin{aligned}
M_U &= 238.050784\,u \\
M_{Th} &= 234.043593\,u \\
m_\alpha &= 4.002602\,u \\
M_U - M_{Th} - m_\alpha &= 0.004589\,u \\
Q = \Delta m \cdot c^2 &= 0.004589 \times 931.502 \\
&= 4.275 \quad MeV
\end{aligned}$$

Disintegration energy

- Most of the energy (Q) is KE of α
- Decay occurs if Q > 0 (energy released)
- Spontaneous decay does not occur if Q < 0

Conservation of momentum (daughter + alpha) and Energy gives …

$$Q = KE(\alpha) + KE(daughter)$$

$$KE(\alpha) = Q \cdot \left(\frac{A-4}{A}\right)$$

$$e.g. \quad {}^{238}U \quad KE(\alpha) = 4.275 \times \left(\frac{234}{238}\right) = 4.2\ MeV$$

Theory of α-decay

- Questions to answer:

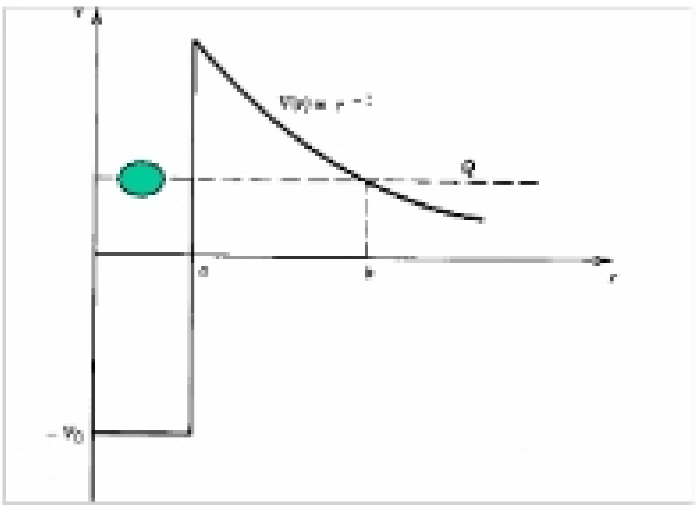
(1) How do α-particles with KE ~ 4 MeV escape the nucleus while incoming α-particles with KE ~ 10 MeV are scattered ?

(2) KE(α) range is ~ 4 to 9 MeV but half-life varies over 24 orders of magnitude !

(nanoseconds to billions of years)

Theory of α-decay

- 1911: Geiger & Nuttall noticed that large Q ↔ short half-life
- 1928: Gamow, Gurney, Condon et al. QM tunnelling through a nuclear potential barrier by α
- The tunnelling model works well, even though it supposes the α to be preformed inside the nucleus.



Krane 8.3