

Chapter Goals:

- Understand average rates of change.
- Understand the ideas leading to instantaneous rates of change.
- Understand the connection between instantaneous rates of change and the derivative.
- Know the definition of the derivative at a point.
- Use the definition of the derivative to calculate derivatives.
- Understand the connection between a position function, a velocity function, and the derivative.
- Understand the connection between the derivative and the slope of a tangent line.

Assignments:

Assignment 02

Assignment 03

Roughly speaking, Calculus describes how quantities change, and uses this description of change to give us extra information about the quantities themselves.

► **Average rates of change:** We are all familiar with the concept of velocity (speed): If you drive a distance of 120 miles in two hours, then your average velocity, or rate of travel, is $120/2 = 60$ miles per hour. In other words, the average velocity is equal to the ratio of the distance traveled over the time elapsed:

$$\text{average velocity} = \frac{\text{distance traveled}}{\text{time elapsed}} = \frac{\Delta s}{\Delta t}.$$

In general, the quantity $\frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$ is called the **average rate of change of y with respect to x** .

Note: Often, a change in a quantity q is expressed by the symbol Δq (you should not think of this as Δ times q , but rather as one quantity!).

Note: Finding average rates of change is important in many contexts. For instance, we may be interested in knowing how quickly the air temperature is dropping as a storm approaches, or how fast revenues are increasing from the sale of a new product.

Note: In this course we use the terms “speed” and “velocity” for the same concept. This is not the case in some other courses. Thus “instantaneous speed” and “instantaneous velocity” have the same meaning, and “average speed” and “average velocity” have the same meaning.

Example 1: A train travels from city A to city B . It leaves A at 10:00 am and arrives at B at 2:30 pm. The distance between the cities is 150 miles. What was the average velocity of the train in miles per hour (mph)? Do you think the train was always traveling at the same speed?

Example 2: A train leaves station A at 8:00 am and arrives at station B at 10:00 am. The train stops at station B for 1 hour and then continues to station C . It arrives at station C at 3:00 pm. The average velocity from A to B was 40 mph and the average velocity from B to C was 50 mph. What was the average velocity from A to C (including stopping time)?

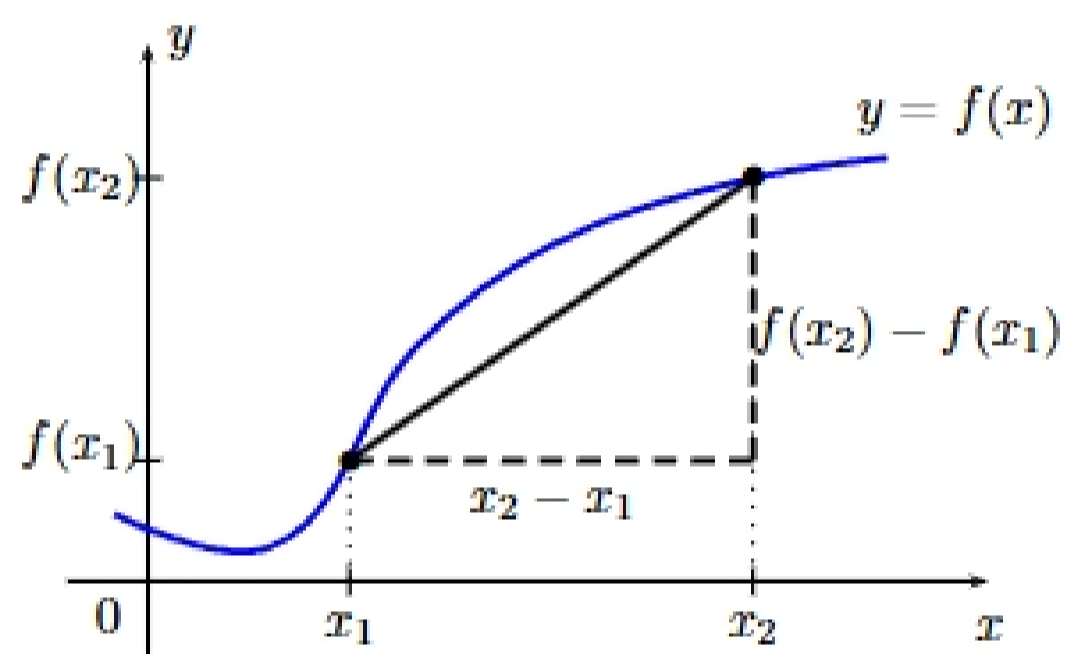
Generally, in computing average rates of change of a quantity y with respect to a quantity x , there is a function that shows how the values of x and y are related.

► **Average rates of change of a function:**

The **average rate of change** of the function $y = f(x)$ between $x = x_1$ and $x = x_2$ is

$$\text{average rate of change} = \frac{\text{change in } y}{\text{change in } x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

The average rate of change is the slope of the **secant line** between $x = x_1$ and $x = x_2$ on the graph of f , that is, the line that passes through $(x_1, f(x_1))$ and $(x_2, f(x_2))$.



Example 3: Find the average rate of change of $g(x) = 2 + 4(x - 1)$ with respect to x as x changes from -2 to 5 . Could you have predicted your answer using your knowledge of linear equations?

Example 4: Find the average rate of change of $k(t) = \sqrt{3t + 1}$ with respect to t as t changes from 1 to 5 .

Example 5: A particle is traveling along a straight line. Its position at time t seconds is given by $s(t) = 2t^2 + 3$. Find the average velocity of the particle as t changes from 0 seconds to 4 seconds.

Example 6: Let $g(x) = \frac{1}{x}$. Find a value for x such that the average rate of change of $g(x)$ from 1 to x equals $-\frac{1}{10}$.

Example 7: Find the average rate of change of $k(t) = t^3 - 5$ with respect to t as t changes from 1 to $1 + h$.

► **Instantaneous rates of change:** The phrase ‘instantaneous rate of change’ seems like an oxymoron, a contradiction in terms like the phrases ‘thunderous silence’ or ‘sweet sorrow’. However, because of your experience with traveling and looking at speedometers, both the concept of *average velocity* and the concept of *velocity at an instant* have an intuitive meaning to you. The connection between the two concepts is that if you compute the average velocity over smaller and smaller time periods you should get numbers that are closer and closer to the speedometer reading at the instant you look at it.

Definition: The instantaneous rate of change is defined to be the result of computing the average rate of change over smaller and smaller intervals.

The following algebraic approach makes this idea more precise.