Chapter 8

Loop Design

8.1 Introduction

This is the first Chapter that deals with design and we will therefore start by some general aspects on design of engineering systems. Design is complicated because there are many issues that have to be considered. Much research and development has been devoted to development of design procedures and there have been numerous attempts to formalize the design problem. It is useful to think about a design problem in terms of specifications, trade-offs, limitations, design parameters. In additions it is useful to be aware of the fact that because of the richness of design problem there are some properties that are captured explicitly by the design method and other properties that must be investigated when the design is completed.

Specifications are an attempt to express the requirements formally. For control systems they typically include: attenuation of load disturbances, measurement noise, process uncertainty and command signal following. Specifications are typically expressed by quantities that capture these features. For control systems the fundamental quantities are typically transfer functions or time responses and specifications are some representative features of these functions. A design fundamentally involves trade-offs. It is also extremely important to be aware of fundamental limitations to avoid unrealistic specifications and impossible trade-offs. Sometimes it is attempted to capture the trade-offs in a single optimization criterion, but such a criterion always includes parameters that have to be chosen. It is often useful to make the trade-offs explicit and to have design parameters that gives the designer control of them. A design method typically focuses on some aspects of a design problem but it is often difficult to capture all aspects of a design problem formally. There will frequently be several properties that must be investigated separately when the design is completed. In control system design it is common practice to verify a design by extensive simulation that may include hardware in the loop. When discussing design in this and the following chapters we have made an attempt to emphasize the fundamental tradeoffs and the design parameters.

In this Chapter we will present a design method which is focused on the loop transfer function of the system. The method is focused on design a feedback controller that satisfies can deal with disturbances and process uncertainty. The response to reference signals can later be dealt with using feedforward which is discussed in the next chapter. The idea of loop shaping is outlined in Section 8.2 which present classical results that were developed when control emerged in the early 1940s. This section also contains some simple compenstators like lead-lag compensation, which is closely related to PID control. Section **??** presents another classical result due to Bode, who developed the concept of an ideal loop transfer function for electronic amplifiers. In the following sections it is shown how the loop transfer function relates to important properties of the closed loop system. Section **??** treats robustness properties. It is shown that the estimates developed earlier can be refined to give more precise information. Two graphical tools, the Hall diagram and the Nichols chart are also introduced. Section **??** deals with load disturbances. An estimate the norm of the transfer function from load disturbances to process output based on the crossover frequency is derived. The result shows that it is typically advantageous to have a high crossover frequency. Section **??** deals with measurement noise. An estimate of the norm of the transfer function from measurement noise to the control signal is developed. It is shown that a high crossover frequency is typically associated with high controller gains at frequencies above the crossover frequency. In Section **??** it is shown that non-minimum phase dynamics imposes severe constraints on the admissible crossover frequencies. Summarizing the findings of this section with the results of Sections **??**, **??**, and **??** we are in a good position to discuss the major trade-offs in the choice of gain crossover frequency. First the choice is severely restricted if process dynamics is not minimum phase. The choice is then governed by a compromise between robustness, attenuation of load disturbances and injection of measurement noise. It turns out that the compromise can be captured in a single diagram. This diagram also indicates the complexity of the controller that is required for different choices. This is shown in Section **??**.

8.2 Compensation

The idea of loop shaping is to find a controller so that the loop transfer function has desired properties. Since the loop transfer function $L = PC$ is the product of the transfer functions of the process and the controller it is easy to see how the loop transfer function is influenced by the controller. Compensation is typically done by successive modifications of the loop transfer function starting with proportional control. If the desired properties cannot be obtained by proportional control the controller is modified by multiplying the controller transfer function by compensating networks having simple transfer functions. The design is conveniently visualized using bode diagrams. Since the Bode diagram is logarithmic multiplication by a compensating network corresponds to additions in the Bode diagram. We start by discussing a desired properties of a loop transfer function, then we present some simple compensators and we end by a few examples. In the following sections we will present a more systematic treatment where we discuss how th different properties of the loop are influenced by the loop transfer function.

Properties of the Loop Transfer Function

The Bode diagram of a typical loop transfer function is shown in Figure 8.1. Recall that the lowest frequency where the loop transfer function has gain 1 is called gain crossover frequency, see Section **??**. This parameter is an important design parameter in loop shaping. When we talk about high and low frequencies in the following we are using the crossover frequency as a reference. The gain curve of a typical loop transfer function is decreasing as shown in Figure 8.1. The behavior at low frequencies determine attenuation of load disturbances and behavior of tracking of low frequency reference signals. The behavior around the crossover frequency determines robustness and sensitivity to modeling errors. The gain margin g_m and the phase margin φ_m are easily visualized in the Bode plot. The behavior at high frequency determines sensitivity to measurement noise.

It is desirable to have high gain at low frequency and rapidly decreasing gain after the gain crossover frequency. Since we require that the closed loop system should be stable, the slope of the gain curve at crossover n_{gc} cannot be too steep at the crossover. However, the phase of a system is related to its gain, and hence it is not possible to independently specify these quantities. In the remainder of this chapter we shall explore some of the limits of this tradeoff between gain and phase through a variety of standard compensator