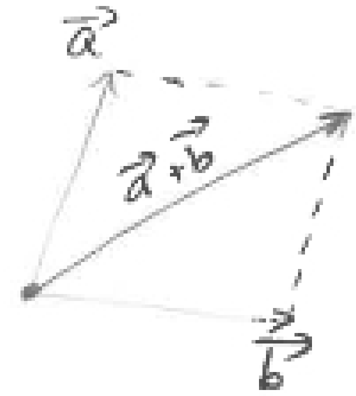
How to draw $\vec{a} + \vec{b}$

⊕ Move $\vec{a}$ and $\vec{b}$ so that they share the same tail



⊕ Complete the parallelogram.

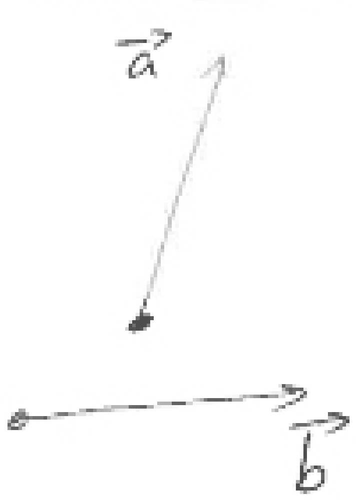
⊕ $\vec{a}+\vec{b}$ lies on the diagonal of the parallelogram. //

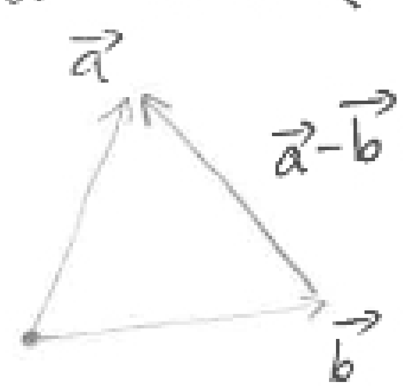
PARALLELOGRAM

- <u>Scalar multiplication</u> $\quad c\vec{a} = \langle ca_1, ca_2, ca_3 \rangle$

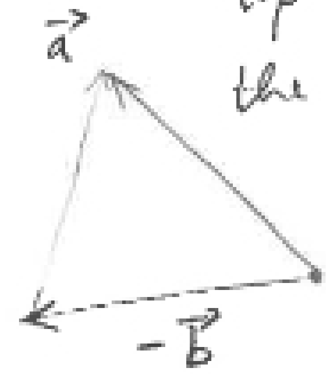
Change direction if $c < 0$
Change magnitude if $c \neq \pm 1$.

- <u>Vector difference</u> $\quad \vec{a} - \vec{b} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$





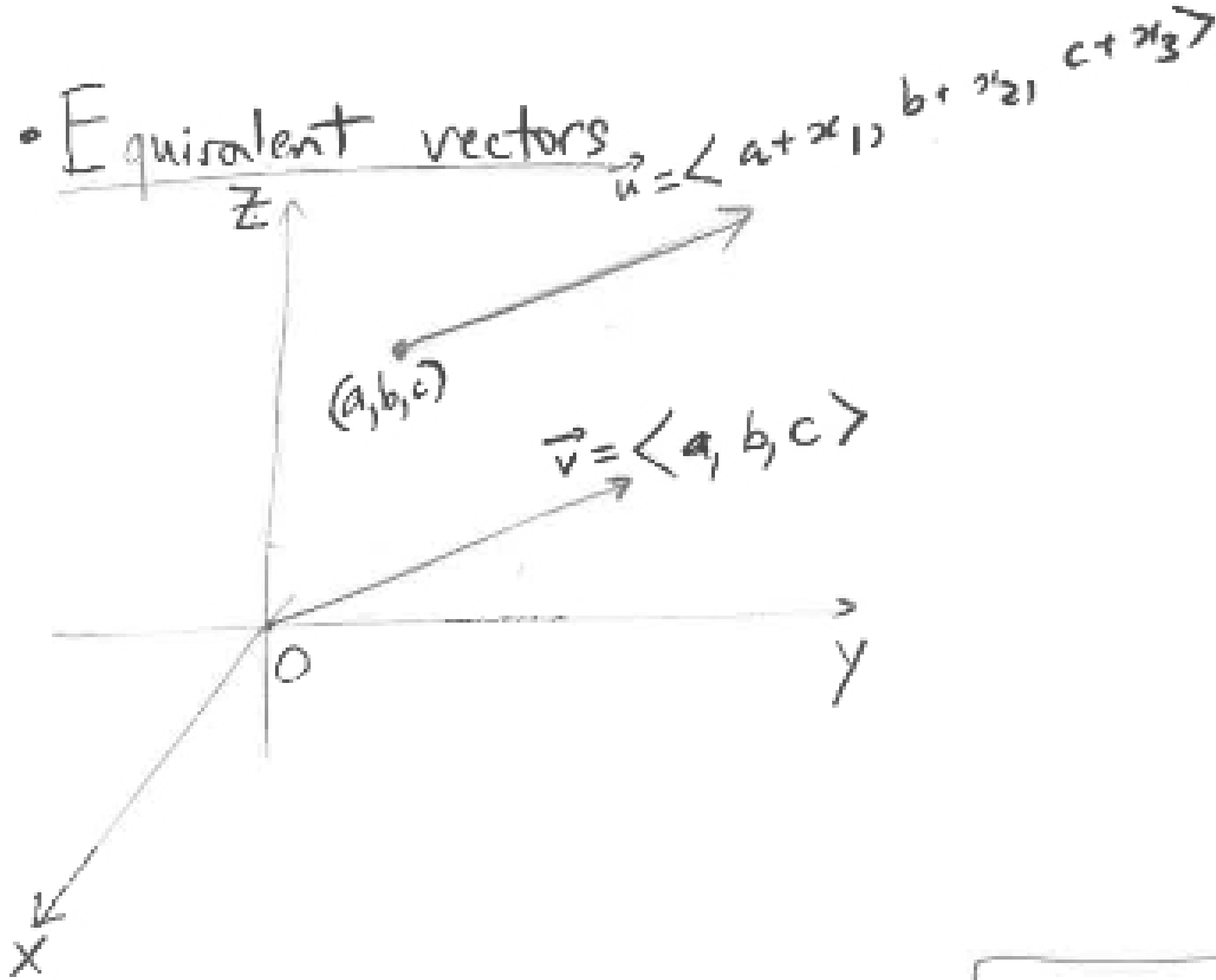
- move $\vec{a}, \vec{b}$ so that they have the same tail
- $\vec{a}-\vec{b}$ has the same tip as $\vec{a}$ and has the tip of $\vec{b}$ as its tail.



Easier way $\quad \vec{a} - \vec{b} = \vec{a} + (-\vec{b})$

- Draw $-\vec{b}$ and apply the triangle law

- **Equivalent vectors**



$\vec{u} = \vec{v}$.
$\vec{v}$ is called a position vector because its tail is at the origin.

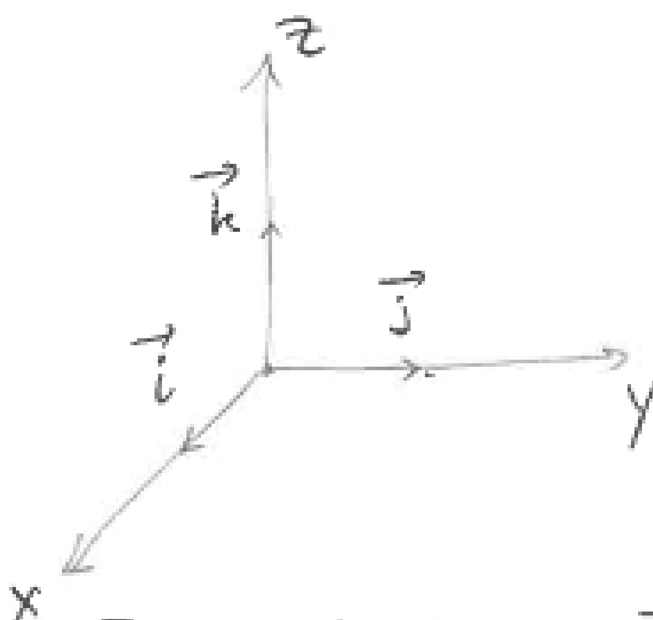
- **Vector length** $|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$.

- **Other properties** $\underbrace{(c + d)}_{\text{scalars}}\vec{a} = c\vec{a} + d\vec{a}$

$c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b}$.

- **Standard basis vectors**

$\vec{i} = \langle 1, 0, 0 \rangle$
$\vec{j} = \langle 0, 1, 0 \rangle$
$\vec{k} = \langle 0, 0, 1 \rangle$



$\vec{a} = \langle a_1, a_2, a_3 \rangle = a_1\vec{i} + a_2\vec{j} + a_3\vec{k}$

Example 1: $\vec{a} = \vec{i} + 5\vec{j} - 2\vec{k}$
$\vec{b} = \vec{i} + 2\vec{j}$

Find $2\vec{a} + 3\vec{b}$ in terms of $\vec{i}, \vec{j}, \vec{k}$.

$$2\vec{a} + 3\vec{b} = 2(\vec{i} + 5\vec{j} - 2\vec{k}) + 3(\vec{i} + 2\vec{j})$$
$$= 5\vec{i} + 16\vec{j} - 4\vec{k}$$

• Unit vector

A vector whose length is 1.

⊕ Example: the standard basis vectors are unit vectors.

⊕ How to convert a nonzero vector to a unit vector:

if $\vec{a} \neq \vec{0}$, then $\dfrac{\vec{a}}{|\vec{a}|}$ is a unit vector, and

$\dfrac{\vec{a}}{|\vec{a}|}$ has the same direction as $\vec{a}$.

Example: $\vec{a} = \langle 1, 0, 1 \rangle$, $|\vec{a}| = \sqrt{1^2 + 0^2 + 1^2} = \sqrt{2}$

Hence, $\dfrac{\vec{a}}{\sqrt{2}} = \left\langle \dfrac{1}{\sqrt{2}}, 0, \dfrac{1}{\sqrt{2}} \right\rangle$ is the unit vector

having the same direction as $\vec{a}$.

(Group activity)

Example 2: $\vec{a} = \langle 4, 5, 2 \rangle$ $\quad \vec{b} = \langle -1, 2, 1 \rangle$

i) Find $\vec{a} - 2\vec{b}$

ii) Compute $|\vec{a} - 2\vec{b}|$

iii) Find a unit vector in the direction of $\vec{a}$

iv) Find a vector in the direction of $\vec{a}$ with length 5.

i) $\vec{a} - 2\vec{b} = \langle 4, 5, 2 \rangle - \langle -2, 4, 2 \rangle = \langle 6, 1, 0 \rangle$.

ii) $|\vec{a} - 2\vec{b}| = \sqrt{6^2 + 1^2 + 0^2} = \sqrt{37}$.

iii) $|\vec{a}| = \sqrt{4^2 + 5^2 + 2^2} = \sqrt{45} = 3\sqrt{5}$. $\quad \dfrac{\vec{a}}{|\vec{a}|} = \left\langle \dfrac{4}{3\sqrt{5}}, \dfrac{\sqrt{5}}{3}, \dfrac{2}{3\sqrt{5}} \right\rangle$

iv) Simple multiply the unit vector in part iii) by 5

$$5\left\langle \frac{4}{3\sqrt{5}}, \frac{\sqrt{5}}{3}, \frac{2}{3\sqrt{5}} \right\rangle = \left\langle \frac{4\sqrt{5}}{3}, \frac{5\sqrt{5}}{3}, \frac{2\sqrt{5}}{3} \right\rangle.$$