10/3/14 MATH 2850

14.6 #49

$T(x,y) = x(e^y + e^{-y})$

$x \approx 2$ maximum possible errors $|dx| = 0.1$

$y \approx \ln 2$ $|dy| = 0.02$

ext value ↓ estimate max possible error in T

$\underbrace{T(x+dx, y+dy) - T(2, \ln 2)}_{\text{error}}$

$= \cancel{T(2,\ln 2)} + T'(2,\ln 2)\begin{pmatrix} dx \\ dy \end{pmatrix} + \text{error} - \cancel{T(2,\ln 2)}$

$= T'(2,\ln 2)\begin{pmatrix} dx \\ dy \end{pmatrix} + \text{error}$

$T'(x,y) = [\,T_x \quad T_y\,]$

$= [\,e^y + e^{-y} \quad x(e^y - e^{-y})\,]$

$T'(2,\ln 2) = [\,e^{\ln 2} + e^{-\ln 2} \quad 2(e^{\ln 2} - e^{-\ln 2})\,]$

$= [\,2 + 1/2 \quad 2(2 - 1/2)\,]$

$= [\,2.5 \quad 3\,]$

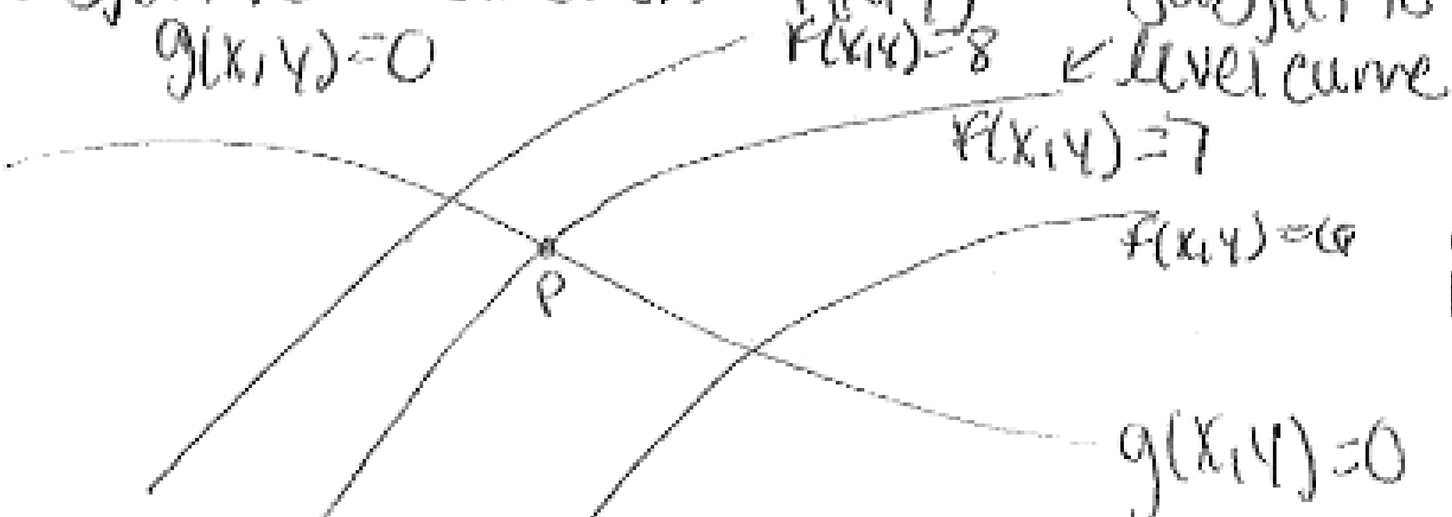
$\approx [2.5 \quad 3]\begin{bmatrix} dx \\ dy \end{bmatrix}$

$= 2.5dx + 3dy$

$= 2.5(0.1) + 3(0.02) = .25 + 0.06 = \boxed{0.31}$

Find extreme values (and where they occur) of the objective function $f(x,y)$ subject to the constraint $g(x,y) = 0$

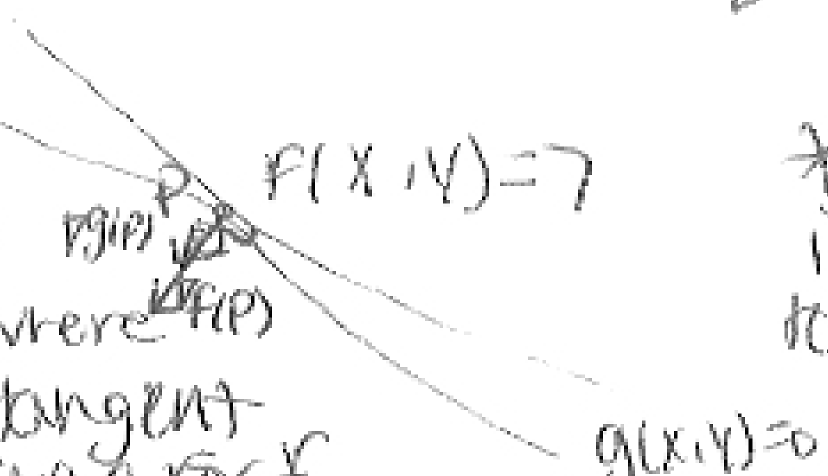
← equation $x + 3y$



* if you cross can't cross a level curve on both sides if it is an extreme value!

↳ P ≠ extrema

* What picture should look like?



Plan
look for points P on the constraint where the constraint is tangent to the level curve for f

* the gradient is normal ($\perp$) to the level surface

$\nabla f(P)$ & $\nabla g(P)$ need to be parallel

* two vectors are parallel when one is just a scaled version of the other

ex: $\langle 1,2\rangle$ $\langle 2,3\rangle$ ← no $2\langle 1,2\rangle = \langle 2,4\rangle$ ← yes

$$\nabla f(P) = \lambda \nabla g(P) \qquad \lambda \nabla f(P) = \nabla g(P)$$

not the same

* always put ~~the~~ λ next to the constraint

$$\begin{cases} \nabla f(P) = \lambda \nabla g(P) \\ g(P) = 0 \end{cases}$$

ex: $f(x,y) = x^2 + y^2$ constraint $\underbrace{x^4 + 2y^4}_{g(x,y)} = 1$

$\nabla f = \langle 2x, 2y\rangle$

$\nabla g = \langle 4x^3, 8y^3\rangle$ $x=?$ $y=?$ $\lambda=?$

$$\begin{cases} 2x = \lambda 4x^3 \\ 2y = \lambda 8y^3 \\ x^4 + 2y^4 = 1 \end{cases}$$

$x=0$
$2y^4 = 1$
$y^4 = 1/2$
$y = \pm\sqrt[4]{1/2}$

$x \neq 0$

$y=0$
$x^4 = 1$
$x = \pm 1$

$y \neq 0$
$2 = \lambda 4x^2$
$2 = \lambda 8y^2$ =
~~λ~~ $4x^2 = \lambda 8y^2$
$x^2 = 2y^2$
$x = \pm\sqrt{2}\sqrt[4]{1/6}$

Back

$(2y^2)^2 + 2y^4 = 1$
$4y^4 + 2y^4 = 1$
$6y^4 = 1$
$y^4 = 1/6$
$y = \pm\sqrt[4]{1/6}$

Points
$(0, \pm\sqrt[4]{1/2})$
$(\pm 1, 0)$
$(\pm\sqrt{2}\sqrt[4]{1/6}, \pm\sqrt[4]{1/6})$

Points	$f(x,y)$		
$(0, \pm\sqrt[4]{1/2})$	$4 0 + \sqrt{1/2}$	$= \frac{1}{\sqrt{2}} \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{6}}$	$\rightarrow \sqrt{3}$ minimum
$(\pm 1, 0)$	1	$= \frac{\sqrt{6}}{\sqrt{6}}$	$\rightarrow \sqrt{6}$
$(\pm\sqrt{2}\sqrt[4]{1/6}, \pm\sqrt[4]{1/6})$	$2\sqrt{1/6} + \sqrt{1/6}$	$= \frac{2}{\sqrt{6}} + \frac{1}{\sqrt{6}} = \frac{3}{\sqrt{6}}$	$\rightarrow 3$ maximum

ex: Find the global extrema of $f(x,y,z) = xyz$ subject to the constraint $\underbrace{x^2+2y^2+3z^2}_{g(x,y,z)} = 6$

Points	$f(x,y,z)$
Some points	0
	$\pm\sqrt{2/3}$

$\nabla f = \langle yz, xz, xy \rangle$

$\nabla g = \langle 2x, 4y, 6z \rangle$

$$\begin{cases} \nabla f = \lambda \nabla g \\ g = 6 \end{cases}$$

$$\begin{cases} yz = \lambda 2x & y=0 \text{ or } z=0 \\ xz = \lambda 4y & x=0 \text{ or } z=0 \\ xy = \lambda 6z & y=0 \text{ or } x=0 \\ x^2+2y^2+3z^2 = 6 \end{cases}$$

$xyz = \lambda 2x^2$
$xyz = \lambda 4y^2$
$xyz = \lambda 6z^2$

$\lambda 2x^2 = \lambda 4y^2 = \lambda 6z^2$
$(2\lambda)x^2 = (2\lambda)2y^2 = (2\lambda)3z^2$
$x^2 = 2y^2 = 3z^2$

$2y^2 = 6$
$y^2 = 3$
$y = \pm\sqrt{3}$

$$\begin{cases} x^2 = 2 \\ 2y^2 = 2 \\ 3z^2 = 2 \end{cases}$$

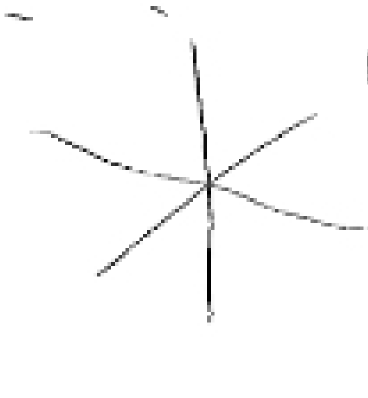
$x = \pm\sqrt{2}$
$y = \pm 1$
$z = \pm\sqrt{2/3}$

global max at 4 points
global min at 4 points

$\pm \; \pm \; \pm$
$2 \cdot 2 \cdot 2$

+	+	+
−	−	+
+	−	−
−	+	−

↑ all plus



octants