9/16/14

A function f is differentiable at P if

$$f(P+\Delta P) = f(P) + f'(P)\,\Delta P + \epsilon(P,\Delta P)$$

where $\dfrac{|\epsilon(P,\Delta P)|}{|\Delta P|} \to 0$ as $|\Delta P| \to 0$

$$f(P+\Delta P) = f(P) + (f_x(P), f_y(P) \dots)\begin{pmatrix}\Delta x\\ \Delta y\\ \vdots\end{pmatrix} + \epsilon(P,\Delta P)$$

$$= f(P) + (f_x(P)\Delta x + f_y(P)\Delta y + \dots) + \epsilon(P,\Delta P)$$

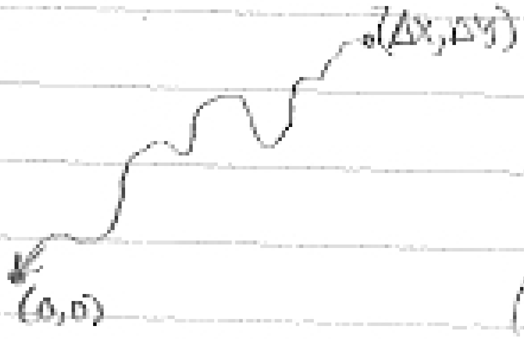
$f(x,y) = x^2y$

$$f(x+\Delta x, y+\Delta y) = (x+\Delta x)^2\,(y+\Delta y)$$
$$= (x^2 + 2x\Delta x + (\Delta x)^2)(y+\Delta y)$$
$$= \underbrace{x^2y}_{f(x,y)} + \underbrace{2x\Delta x y}_{f_x(x,y)\Delta x} + (\Delta x)^2 y + \underbrace{x^2\Delta y}_{f_y(x,y)\Delta y} + 2x\Delta x\Delta y + (\Delta x)^2\Delta y$$

$$= \underbrace{x^2y}_{f(x,y)} + \underbrace{(2xy\Delta x + x^2\Delta y)}_{(f_x\ f_y)\binom{\Delta x}{\Delta y}} + \underbrace{(\Delta x\Delta x y + 2x\Delta x\Delta y + (\Delta x)^2\Delta y)}_{\epsilon(x,y,\Delta x,\Delta y)}$$

Show $\lim\limits_{\left|\binom{\Delta x}{\Delta y}\right| \to 0} \dfrac{|y\Delta x\Delta x + 2x\Delta x\Delta y + \Delta x\Delta x\Delta y|}{\sqrt{\Delta x\Delta x + \Delta y\Delta y}}$

either notation: $(\Delta x,\Delta y) \to (0,0)$



$\Delta x = r\cos\theta$

$\Delta y = r\sin\theta$

$(\Delta x)^2 + (\Delta y)^2 = r^2$

$$= \lim_{r\to 0} \left| y r^{\cancel{2}}\cos^2\theta + 2xr^{\cancel{2}}\cos\theta\sin\theta + r^{\cancel{3}\,2}\cos^2\theta\sin\theta \right|$$

$$= \lim_{r\to 0} r\left|y\cos^2\theta + 2x\cos\theta\sin\theta + r^2\cos^2\theta\sin\theta\right| = 0$$

$f_x(xy) = 2xy$

$f_y(x,y) = x^2$

$f(x+\Delta x, y+\Delta y)$

$f(x+\Delta x, y) = (x+\Delta x)^2 y = (x^2 + 2x\Delta x + \Delta x\Delta x)y$

$= \underbrace{x^2y}_{f(x,y)} + (2xy\Delta x) + (\Delta x\Delta x\Delta y)$

(x,y)

$g(x+\Delta x) = g(x) + g'(x)\Delta x + \epsilon(\ ,\ )$

$$\underbrace{\begin{array}{l} f_x(x,y) = 2xy \Rightarrow \text{hold } y \text{ as constant \& derive } x^2y \\ f_y(x,y) = x^2 \rightarrow \text{hold } x \text{ as constant \& derive } x^2y \end{array}}_{\text{partial derivatives}}$$

$f_x(x,y) = \frac{\partial f}{\partial x} = f_x$ & $f_y(x,y) = f_y = \frac{\partial f}{\partial y}$

$f(x,y) = x^3y^2 + 7xy^3 + x^2$

$$\left.\begin{array}{l} f_x(x,y) = 3x^2y^2 + 7y^3 + 2x \\ f_y(x,y) = 2yx^3 + 21y^2x + 0 \end{array}\right\} \text{partial derivatives of}$$

$f'(xy) = (f_x\ f_y) = (3x^2y^2 + 7y^3 + 2x\ ,\ 2yx^3 + 21y^2x) = Df$

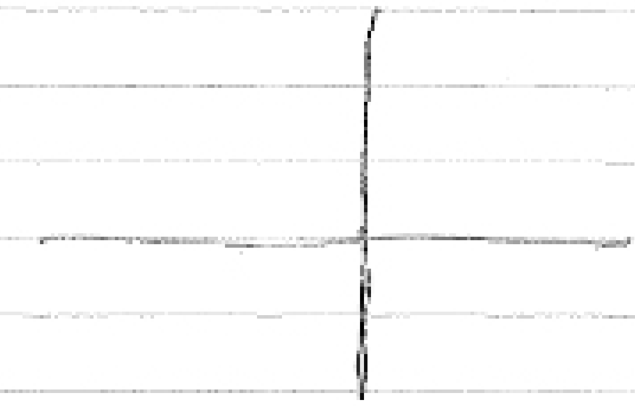
$f'(4,7) = (3(4)^2(7)^2 + 7(7)^3 + 2(4)\ ,\ 2(7)(4)^3 + 21(7)^2(4))$

$= (4761\ ,\ 5012)$

Theorem: if the partials of f exist and are continuous in an open disk around P the f is differentiable at P

EX of bad function....

$$f(x,y) = \begin{cases} 0 & \text{if } xy \neq 0 \\ 1 & \text{if } xy = 0 \end{cases}$$



Compute $f'(0,0)$

but first the partials:

$f_x(0,0) = 0$
$f_y(0,0) = 0$

$$f(0+\Delta x, 0+\Delta y) = f(0,0) + f_x(0,0)\Delta x + f_y(0,0)\Delta y + \epsilon(0,0,\Delta x,\Delta y)$$
$$= 1 + 0 + 0 + \epsilon(0,0,\Delta x,\Delta y)$$

Want: $\dfrac{|\epsilon(0,0,\Delta x,\Delta y)|}{|\Delta x \Delta y|} = 0$ as $(\Delta x,\Delta y) \to (0,0)$

$f(\Delta x,\Delta y) - 1 = \epsilon(0,0,\Delta x,\Delta y$ so..

$$\frac{|f(\Delta x,\Delta y) - 1|}{\sqrt{\Delta x^2 + \Delta y^2}} \quad \text{as } (\Delta x,\Delta y) \to (0,0)$$

that goes to $\frac{1}{0}$, undefined. No derivative at origin even though there are partials.