

Matched, Lossless, Reciprocal Devices

As we discussed earlier, a device can be **lossless** or **reciprocal**. In addition, we can likewise classify it as being **matched**.

Let's examine **each** of these three characteristics, and how they relate to the **scattering matrix**.

Matched

A matched device is another way of saying that the **input impedance** at each port is **equal to Z_0** when **all other** ports are terminated in matched loads. As a result, the **reflection coefficient** of each port is **zero**—no signal will be come out of a port if a signal is incident on that port (and only that port).

In other words, we want:

$$V_m^- = S_{mm} V_m^+ = 0 \quad \text{for all } m$$

a result that occurs when:

$$S_{mm} = 0 \quad \text{for all } m \text{ if matched}$$

We find therefore that a matched device will exhibit a scattering matrix where all **diagonal elements are zero**.

Therefore:

$$\bar{\mathbf{S}} = \begin{bmatrix} 0 & 0.1 & j0.2 \\ 0.1 & 0 & 0.3 \\ j0.2 & 0.3 & 0 \end{bmatrix}$$

is an example of a scattering matrix for a **matched**, three port device.

Lossless

Recall for a lossless device, all of the power that delivered to each device port must eventually finds its way **out!**

In other words, power is not **absorbed** by the network—no power to be **converted to heat!**

Consider, for example, a **four-port** device. Say a signal is incident on port 1, and that **all other ports are terminated**. The power **incident** on port 1 is therefore:

$$P_1^+ = \frac{|V_1^+|^2}{2Z_0}$$

while the power **leaving** the device at each port is:

$$P_m^- = \frac{|V_m^-|^2}{2Z_0} = \frac{|S_{m1}V_1^-|^2}{2Z_0} = |S_{m1}|^2 P_1^+$$

The **total** power leaving the device is therefore:

$$\begin{aligned} P_{out} &= P_1^- + P_2^- + P_3^- + P_4^- \\ &= |S_{11}|^2 P_1^+ + |S_{21}|^2 P_1^+ + |S_{31}|^2 P_1^+ + |S_{41}|^2 P_1^+ \\ &= (|S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 + |S_{41}|^2) P_1^+ \end{aligned}$$

Note therefore that if the device is **lossless**, the output power will be **equal** to the input power, i.e., $P_{out} = P_1^+$. This is true **only** if:

$$|S_{11}|^2 + |S_{21}|^2 + |S_{31}|^2 + |S_{41}|^2 = 1$$

If the device is lossless, this will likewise be true for each of the **other** ports:

$$|S_{12}|^2 + |S_{22}|^2 + |S_{32}|^2 + |S_{42}|^2 = 1$$

$$|S_{13}|^2 + |S_{23}|^2 + |S_{33}|^2 + |S_{43}|^2 = 1$$

$$|S_{14}|^2 + |S_{24}|^2 + |S_{34}|^2 + |S_{44}|^2 = 1$$

We can state in general then:

$$\sum_{m=1}^N |S_{mn}|^2 = 1 \quad \text{for all } n$$

In fact, it can be shown that a lossless device will have a **unitary** scattering matrix, i.e.: