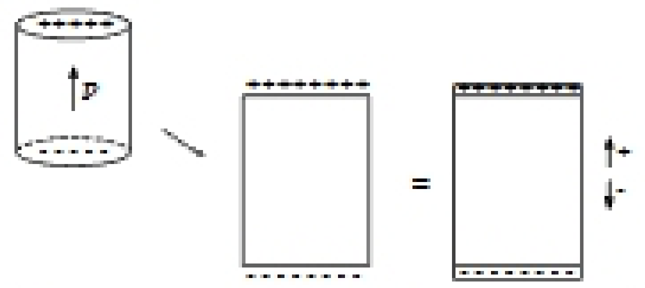
Today in Physics 217: dielectrics

- Finish things from last lecture
- Dielectrics
- Electric polarization and bound charge
- Calculation of field and potential from uniformly-polarized objects



Dielectric materials

Solids are generally composed of neutral atoms and molecules, some of which have built-in, permanent dipole moments and some of which are simply polarizable. For non-conducting solids,

- there is zero dipole moment on large scales, since the orientation of permanent dipoles is generally random.
- immersion in an electric field polarizes atoms and molecules, and tends to align their permanent dipole moments.
- this **polarization** is characterized by a dipole moment per unit volume, in the same direction as the applied field.

Non-conducting solids that can be polarized in this way are called **dielectrics**.

Electric polarization and bound charge

The electric polarization $\mathbf{P}$ is a vector quantity:

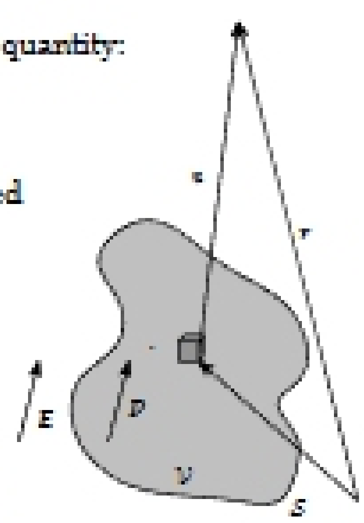
$$\mathbf{P} = \frac{d}{d\tau}\mathbf{p} \qquad \mathbf{p} = \int \mathbf{P}\, d\tau'$$

The potential from a lump of polarized matter is

$$V = \frac{\hat{\boldsymbol{\imath}} \cdot \mathbf{p}}{\imath^2} = \int_{\mathcal{V}} \frac{\hat{\boldsymbol{\imath}} \cdot \mathbf{P}}{\imath^2}\, d\tau'$$

Now, $\frac{\hat{\boldsymbol{\imath}}}{\imath^2} = -\nabla_{\imath}\left(\frac{1}{\imath}\right) = -\nabla\left(\frac{1}{\imath}\right)$

$= \nabla'\left(\frac{1}{\imath}\right)$ as we saw long time ago (lecture, 18 September)



Electric polarization and bound charge (continued)

(Reminder: take the Cartesian components of $\mathfrak{r} = r - r'$ to be X,Y,Z, those of r to be x,y,z, those of r' to be x',y',z'. Then

$$\nabla = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} = \begin{pmatrix} \frac{\partial X}{\partial x}\frac{\partial}{\partial X} \\ \frac{\partial Y}{\partial y}\frac{\partial}{\partial Y} \\ \frac{\partial Z}{\partial z}\frac{\partial}{\partial Z} \end{pmatrix} = \begin{pmatrix} \frac{\partial}{\partial X} \\ \frac{\partial}{\partial Y} \\ \frac{\partial}{\partial Z} \end{pmatrix} = \nabla_{\mathfrak{r}} \qquad \nabla' = \begin{pmatrix} \frac{\partial}{\partial x'} \\ \frac{\partial}{\partial y'} \\ \frac{\partial}{\partial z'} \end{pmatrix} = \begin{pmatrix} \frac{\partial X}{\partial x'}\frac{\partial}{\partial X} \\ \frac{\partial Y}{\partial y'}\frac{\partial}{\partial Y} \\ \frac{\partial Z}{\partial z'}\frac{\partial}{\partial Z} \end{pmatrix} = \begin{pmatrix} -\frac{\partial}{\partial X} \\ -\frac{\partial}{\partial Y} \\ -\frac{\partial}{\partial Z} \end{pmatrix} = -\nabla_{\mathfrak{r}}$$

So

$$V = \int P \cdot \nabla'\left(\frac{1}{\mathfrak{r}}\right) d\tau'$$

Electric polarization and bound charge (continued)

Let's integrate this by parts, using product rule #5:

$$\nabla \cdot (fA) = f\nabla \cdot A + A \cdot (\nabla f)$$

$$V = \int_{\mathcal{V}} P \cdot \nabla'\left(\frac{1}{\mathfrak{r}}\right) d\tau' = \int_{\mathcal{V}} \nabla' \cdot \left(\frac{P}{\mathfrak{r}}\right) d\tau' - \int_{\mathcal{V}} \frac{1}{\mathfrak{r}} \nabla' \cdot P \, d\tau'$$

$$= \oint_{\mathcal{S}} \frac{P}{\mathfrak{r}} \cdot da' - \int_{\mathcal{V}} \frac{1}{\mathfrak{r}} \nabla' \cdot P \, d\tau' \quad .$$

Define the surface and volume bound charge densities:

$$\sigma_b = P \cdot \hat{n} \qquad (\hat{n} = \text{ outward normal of } \mathcal{S})$$

$$\rho_b = -\nabla \cdot P$$

Electric polarization and bound charge (continued)

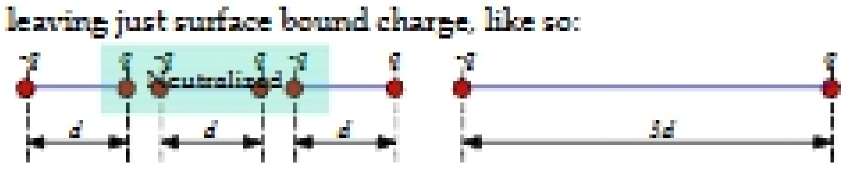
Then the potential takes on a familiar form:

$$V = \oint_{\mathcal{S}} \frac{\sigma_b}{\mathfrak{r}} da' - \int_{\mathcal{V}} \frac{\rho_b}{\mathfrak{r}} d\tau' \quad .$$

Bound charge is the charge displaced by the field into dipolar form. Note that for a *uniform* (constant) polarization, the bound volume charge density is zero:

$$\rho_b = -\nabla \cdot P = 0$$

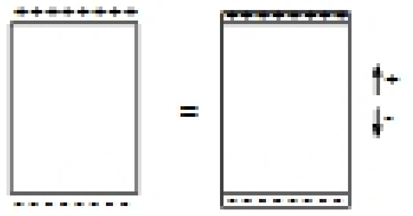
leaving just surface bound charge, like so:



Calculation of field and potential from uniformly-polarized objects

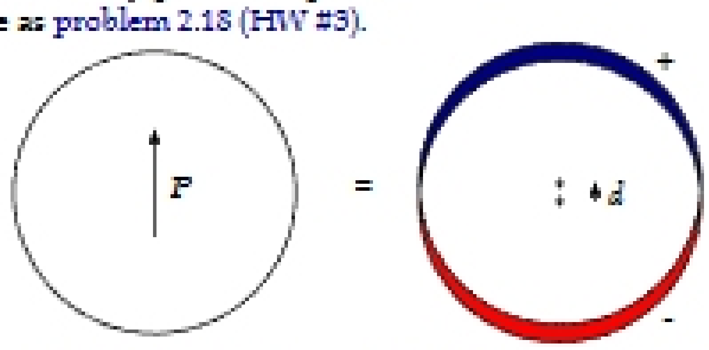
You'll be please to know that you've already made major progress toward calculations involving uniformly-polarized objects, because you can make them by superposition of uniformly-charged objects. Consider a uniformly polarized cylinder:

Same as two uniform, oppositely charged cylinders, displaced infinitesimally along the axis (see problem 2.27, HW#4).



Calculation of field and potential from uniformly-polarized objects (continued)

Or a uniformly-polarized sphere, which would work out the same as problem 2.18 (HW #3).



Uniformly-polarized sphere, radius R.

Two uniformly-charged spheres, density $\pm\rho$, displaced by d.

Calculation of field and potential from uniformly-polarized objects (continued)

We found in that problem that the field in the overlap region (or, if you like, inside the polarized sphere) is

$$\mathbf{E} = -\frac{4\pi\rho}{3}\mathbf{d} = -\frac{q\mathbf{d}}{R^3} = -\frac{\mathbf{p}}{R^3} = -\frac{4\pi\mathbf{P}}{3} \quad .$$

Outside, the field is just that of a simple dipole, since both spheres act like point charges located at their centers:

$$\mathbf{E} = \frac{p}{r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right) = \frac{4\pi R^3 P}{r^3}\left(2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right)$$

(z axis is along $\mathbf{P}$). Note that the (in principle infinitesimal) displacement distance d drops out of the problem when everything is expressed in terms of polarization.