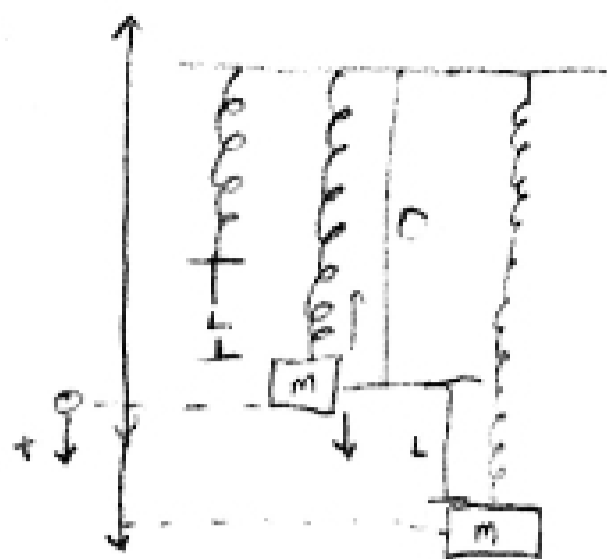


3.7 Hooke's Law: Restoring Force =  $-Ky$



$$mg = Ky$$

$$\frac{mg}{y} = K$$

$$a: My'' = -Ky$$

$$My'' + Ky = 0$$

$$m\tau^2 + k = 0$$

$$\tau^2 = -\frac{k}{m}$$

$$\tau = \pm \sqrt{\frac{k}{m}} i$$

$$A \cos \omega t + B \sin \omega t = A \cos(\omega t + \delta)$$

or

$$y = A \cos(\omega t + \delta)$$

$$A = \sqrt{a^2 + b^2}$$

$$= A(\cos \omega t \cos \delta - \sin \omega t \sin \delta)$$

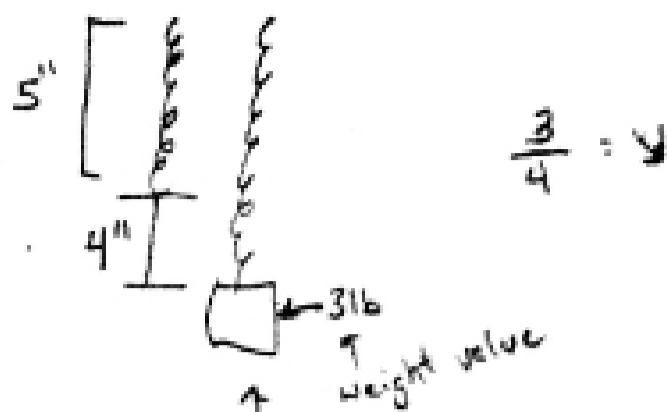
$$A \cos \delta = a \quad A \sin \delta = b$$

$$\cos \delta = \frac{a}{A} \quad \sin \delta = \frac{b}{A}$$

$$\cos^2 \delta + \sin^2 \delta = 1 = \frac{a^2}{A^2} + \frac{b^2}{A^2}$$

$$A = \sqrt{a^2 + b^2}$$

Make sure to convert units.



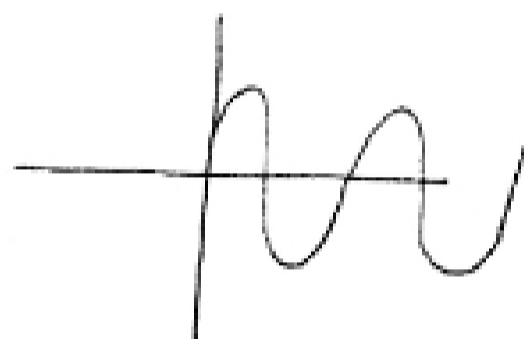
$$\frac{F}{\Delta L} = k$$

$$\text{if it was } 2kg \rightarrow \frac{2kg(9.8)}{4} = \frac{9.8}{2} \text{ (m/sec)}^2$$

$$\omega = \sqrt{\frac{k}{m}}$$

$$y = A \cos \omega t + B \sin \omega t$$

graph



Example:

$$my'' + ky = 0$$

A spring with a mass of 2kg has a normal length of 0.5m. 25.6N is required to stretch the spring to 0.7m, if release find position function

$$m = 2 \quad g = 9.8 \quad F = 25.6N$$

$$y = (0.7 - 0.5) = 0.2$$

$$k = \frac{25.6N}{0.2} = 128$$

$$2y'' + 128y = 0$$

$$y'' + 64y = 0$$

$$\tau = \pm 8i$$

$$y = a \cos 8t + b \sin 8t$$

$$0.2 = a \cos(0) + b \sin(0)$$

$$0.2 = a(1)$$

$$a = 1/5$$

$$y'(0) = 0$$

$$y' = -8a \sin 8t + 8b \cos 8t$$

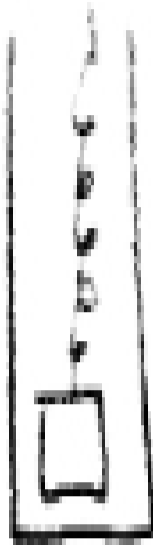
$$y' = 8b(1)$$

$$0 = 8b$$

$$b = 0$$

$$y = \frac{1}{5} \cos 8t$$

# Damping a spring



damp force  $= -CV$   
 $= -Cy'$

$$my'' = -ky - Cy'$$

$$my'' + Cy' + ky = 0$$

$$m\lambda^2 + C\lambda + k = 0$$

$$\lambda = \frac{-C \pm \sqrt{C^2 - 4m(k)}}{2m}$$

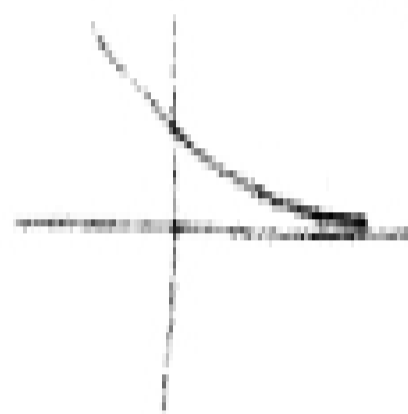
Both be negative

$$C^2 - 4mk > 0$$

$$y = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t}$$

graph

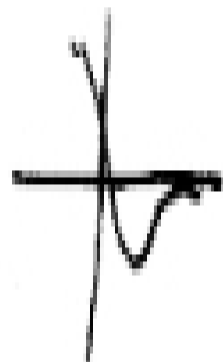
over damped



critically damped  
 if the solution is zero

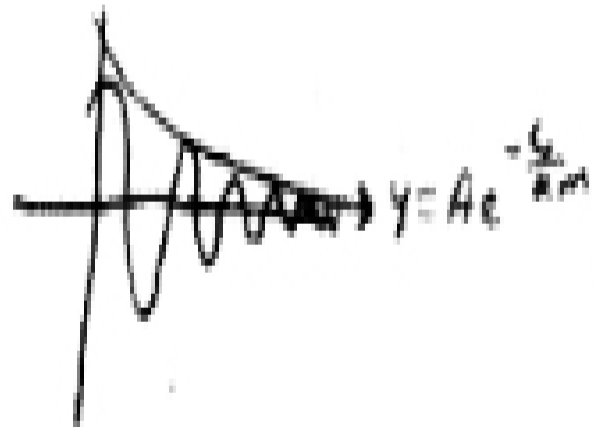
$$C^2 - 4mk = 0$$

$$y = C_1 e^{\lambda t} + C_2 t e^{\lambda t}$$



$$C^2 - 4mk < 0 \quad \text{under damped}$$

$$y = e^{-\frac{C}{2m}t} (a \cos \omega t + b \sin \omega t)$$



HW. Pg 203 3.7  
 1-11 odd

b)