$y'' - 8y' + 12y = g(t)$

$y(0) = 0 \quad y'(0) = 0$

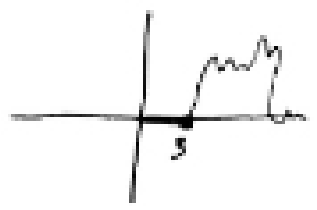
$$g(t) = \begin{cases} 0 & \text{if } 0 \le t < 3 \\ 1 & \text{if } 3 \le t < 10 \\ 0 & \text{if } t \ge 10 \end{cases} \qquad u_3(t) - u_{10}(t)$$

$y'' - 8y' + 12y = u_3(t) - u_{10}(t)$

$y(0), y'(0)$ $1'' - 8y' + 12 = 0$ if $0 \le t < 3$

$= 1$ if $3 \le t < 10$

$y(3), y'(3) =$ $= 0$ if $t \ge 10$



$\mathcal{L}\{y'' - 8y' + 12y = u_3(t) - u_{10}(t)\}$ $\quad s^2\mathcal{L}(y) - sy(0) - y'(0) - 8[s\mathcal{L}(y) - y(0)] + 12\mathcal{L}y =$

$\frac{e^{-3s} - e^{-10s}}{s}$

$$\mathcal{L}(y) = \frac{e^{-3s} - e^{-10s}}{s(s^2 - 8s + 12)} = (e^{-3s} - e^{-10s})\underbrace{\left(\frac{1}{s(s-2)(s-6)}\right)}_{H(s)} \longrightarrow \frac{a}{s} + \frac{b}{s-2} + \frac{c}{s-6}$$

$1 = a(s-2)(s-6) + b(s)(s-6) + c(s)(s-2)$

$a(s^2 - 8s + 12) + b(s^2 - 6s) + c(s^2 - 2s)$

$as^2 - 8as + 12a + bs^2 - 6bs + cs^2 - 2sc$

$0 = as^2 + bs^2 + cs^2$

$0 = -8sa - 6b - 2c$

$1 = 12a \quad a = 1/12$

$b = -1/8$

$c = 1/24$

$$\mathcal{L}(y) = e^{-3s}H(s) - e^{-10s}H(s)$$

what is this?

$$H(s) = \frac{1}{12}\left(\frac{1}{s}\right) - \frac{1}{8}\left(\frac{1}{s-2}\right) + \frac{1}{24}\left(\frac{1}{s-6}\right)$$

$$h(t) = 1/12 - 1/8\, e^{2t} + 1/24\, e^{6t}$$

Solution: $y = u_3(t)h(t-3) - u_{10}(t)h(t-10)$

answer

$$y = e^{2t} + e^{6t} + u_3(t)\left(\frac{1}{12} - \frac{1}{8}e^{2t-6} + \frac{1}{24}e^{6t-18}\right) - u_{10}(t)\left(\frac{1}{12} - \frac{1}{8}e^{2t-20} + \frac{1}{24}e^{6t-60}\right)$$

$y'' - 5y' + 6y = g(t)$

$y(0) = 0 \quad y'(0) = 0$

$$g(t) = \begin{cases} 1 & \text{if } t < 2 \\ 2 & \text{if } 2 \le t < 4 \\ 0 & \text{if } t \ge 4 \end{cases}$$

$$u_M(t) = \begin{cases} 0 & \text{if } 0 < t < M \\ 1 & \text{if } t \ge M \end{cases}$$

$\mathcal{L}\{y'' - 5y' + 6y = 1 + u_2(t) - 2u_4(t)\}$

$s^2\mathcal{L}(y) - s\cancel{y(0)} + \cancel{y'(0)} - 5[s\mathcal{L}y - \cancel{y(0)}] + 6\mathcal{L}y = \frac{1 + e^{-2s} - 2e^{-4s}}{s}$

$$\mathcal{L}(y) = \frac{1 + e^{-2s} - 2e^{-4s}}{s(s-2)(s-3)} = \left(\frac{1}{s(s-2)(s-3)}\right)(1 + e^{-2s} - 2e^{-4s})$$

$1(H(s)) + \underbrace{e^{-2s}(H(s))}_{\mathcal{L}(h(t))} - 2e^{-4s}(H(s))$

$\frac{a}{s} + \frac{b}{s-2} + \frac{c}{s-3} = \frac{1}{s(s-2)(s-3)}$ $\quad a(s^2 - 5s + 6) + b(s^2 - 3s) + c(s^2 - 2s) = 1$

$as^2 - 5as + 6a + bs^2 - 3sb + cs^2 - 2sc = 1$

$as^2 + bs^2 + cs^2 = 0$ $\quad a = 1/6$

$-5sa - 3sb - 2sc = 0$ $\quad b = -1/2$

$6a = 1$ $\quad c = 1/3$

$1/6\left(\frac{1}{s}\right) - 1/2\left(\frac{1}{s-2}\right) + 1/3\left(\frac{1}{s-3}\right)$

$h(t) = 1/6 - 1/2\, e^{2t} + 1/3\, e^{3t}$

HW: 6.4

1-7 odd

$$y = 1/6 - 1/6\, e^{2t} + 1/3\, e^{3t} + u_2(t)\left(\frac{1}{6} - \frac{1}{2}e^{2t-4} + \frac{1}{3}e^{3t-6}\right) - 2u_4\left(1/6 - 1/2\, e^{2t-8} + 1/3\, e^{3t-12}\right)$$