5.4 Differentiation of Exponential Functions

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using Tan’s 5th edition Applied Calculus for the managerial , life, and social sciences text

Let's consider the derivative of the exponential function. Going back to our limit definition of the derivative:

$$\frac{d}{dx}\left[f(x)\right]=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\frac{d}{dx}\left(e^{x}\right)=\lim_{h\to 0}\frac{e^{x+h}-e^{x}}{h}$$

First rewrite the exponential using exponent rules.

$$=\lim_{h\to 0}\frac{e^{x}e^{h}-e^{x}}{h}$$

Next, factor out e^x.

$$=\lim_{h\to 0}\frac{e^{x}\left(e^{h}-1\right)}{h}$$

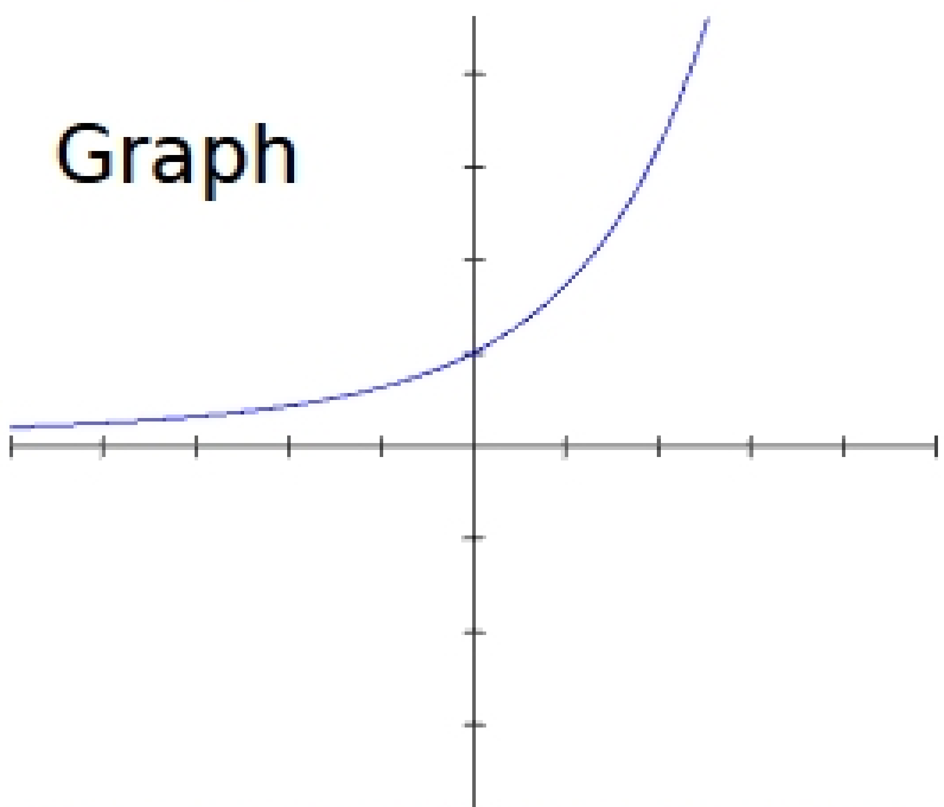
Since e^x does not contain h, we can move it outside the limit.

$$=e^{x}\lim_{h\to 0}\frac{e^{h}-1}{h}$$

Substituting h=0 in the limit expression results in the indeterminate form $\frac{0}{0}$, thus we will need to determine it.

We can look at the graph of $f(x) = \frac{e^x - 1}{x}$ and observe what happens as x gets close to 0. We can also create a table of values close to either side of 0 and see what number we are closing in on.

Graph



Table

x	-.1	-.01	-.001	.001	.01	.1
y	.95	.995	.999	1.0005	1.005	1.05

At x = 0, f(0) appears to be 1. As x approaches 0, y approaches 1.