General Physics - E&M (PHY 1308) Lecture Notes

Lecture 018: The Magnetic Dipole Moment

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The behavior of current loops in external magnetic fields

There is one more important phenomenon to consider: what happens when a current loop is itself exposed to a magnetic field? This is important because this problem is the basis of electric motors, which are ubiquitous in the world: from fans that cool a room, to hybrid gas-electric cars. The basis of such a motor is the force exerted on a loop of a loop of current when it is immersed in an external magnetic field.

Consider a simple geometry of a square loop. Consider also the simple case of a uniform magnetic field into which the loop can be placed. Remember that the force exerted by an external magnetic field on a current is:

$$\vec{F} = I\vec{L} \times \vec{B}$$

- What happens if we place the loop in the field so that originally the plane of the loop is perpendicular to the field?

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\oint ds = 2 \pi
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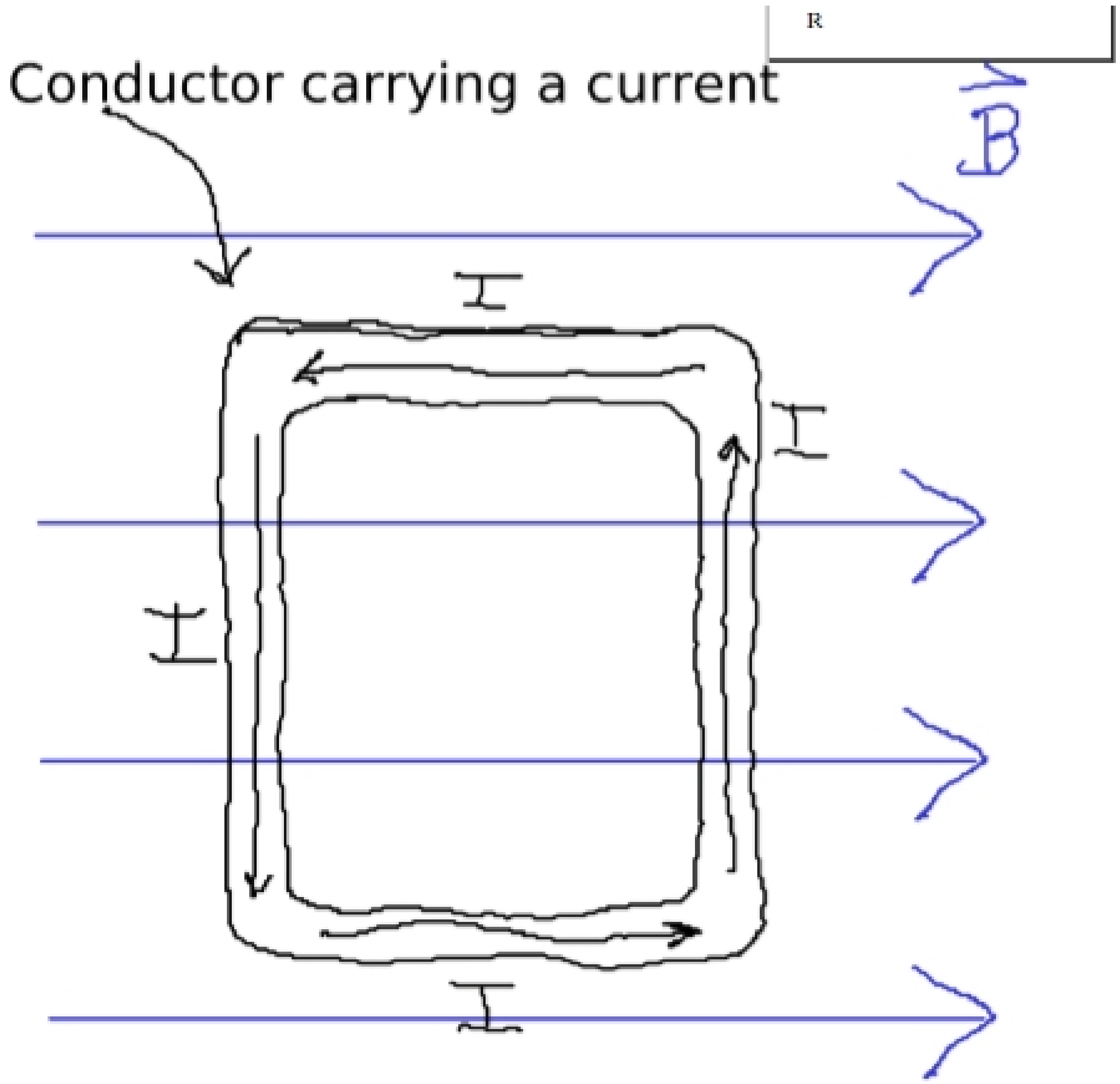

In the drawing above, we can think about what happens by breaking the problem into 4 currents, each one moving in a different direction but all the same magnitude.

- The top current is moving from right to left, and is anti-parallel to the magnetic field. Thus its cross-product magnitude is

$$|I_{top}\vec{L} \times \vec{B}| = ILB\sin\theta = ILB\sin\pi = 0.$$

  Therefore, there is no force on the top segment of the current.
- The bottom current leads us to the same conclusion: it is parallel to the magnetic field, and so the magnitude of its cross-product is also zero.
- Let's consider the left, downward-pointing current. Now we have a current moving at a right-angle to the magnetic field (actually, at $\theta = -\pi/2$). We then expect that the force will point OUT of the page (out of the plane of the blackboard). We can also apply the right-hand rule to see this: stick our fingers in the direction of the current, curl the fingers toward the magnetic field, and let our thumb tell us the direction of the

force - then we see that it points outward. Thus the left-hand current experiences an *outward-pointing force due to the magnetic field*.

- Finally, we consider the right-hand, upward-pointing current. The only difference between it and the left-hand current is the direction of the current. Since the direction is up, the sign of the resulting force reverses (we now have $\sin \pi/2$ instead of $\sin \pi/2$ in the cross-product magnitude), and the right-hand current *experiences a magnetic force pointing into the page/blackboard*.

So what have we learned? We have learned that a loop of current that begins fully parallel to the magnetic field will experience equal but opposite forces on the left and right side. This will cause a *rotation* - and thus there is a *torque* resulting from the force exerted by the magnetic field.

Recall that the definition of a torque is:

$$\vec{\tau} = \vec{r} \times \vec{F}$$

where $\vec{r}$ is a vector that points from the axis of rotation to the point where the force is exerted. In the case above, this is a vector from the vertical bisecting axis of the loop to the right or left-hand sides of the loop.

In our example above, the torque vector points vertically upward (again, use the right-hand rule for the cross-product to verify this: point your fingers in the direction of $\vec{r}$, curl your fingers toward $\vec{F}$, and your thumb indicates the direction of the torque vector.)

More loops

What if we then wrap the wire twice, so that there are two overlapping loops of wire that start out in the plane of the paper? Now we have twice the current on each side of the loop (each wire carries current I, but there are two wires one each side now). As a result, we expect the magnetic force to double.

If we wind 3 times, we expect the force to triple. And so on. For N loops, the total current is NI.