

## Lecture-3

### Search Directions

## Homework Due 1/16/01

- 2.1, 2.2, 2.3, 2.8, 2.13, 2.14

## Rate of Convergence

Definition : Suppose  $\{p_n\}_{n=0}^{\infty}$  is a sequence that converges to  $p$  and that  $e_n = p_n - p$

$$\lim_{n \rightarrow \infty} \frac{|p_{n+1} - p|}{|p_n - p|^\alpha} = \lim_{n \rightarrow \infty} \frac{|e_{n+1}|}{|e_n|^\alpha} = \lambda$$

then the seq is said to converge to  $p$  of order  $\alpha$  with asymptotic error constant  $\lambda$ .

$\alpha = 1$ , linear

$\alpha = 2$ , quadratic

$\alpha = 1$ , and  $\lambda = 0$ , superlinear

## Problem

$$\min_x f(x)$$

## Definitions

A point  $x^*$  is a stationary point if  $f'(x^*) = 0$

A point  $x^*$  is a global minimizer if  $f(x^*) \leq f(x) \quad \forall x$

A point  $x^*$  is a local minimizer if there is a neighborhood  $N$  s.t.  
 $f(x^*) \leq f(x) \quad \forall x \in N$

A point  $x^*$  is a strict local minimizer if  
there is a neighborhood  $N$  s.t.

$$f(x^*) < f(x) \quad \forall x \in N, x \neq x^*$$

if  $\nabla f(x^*) = 0$ , but  $x^*$  is neither a minimum nor  
a maxima, it is called a saddle point.

## First Order necessary conditions

If  $x^*$  is a local minimizer and  $f$   
is continuously differentiable in an  
open neighborhood of  $x^*$ , then  $\nabla f(x^*) = 0$ .