

1 Inductive Reasoning (30 points)

Write a computer program to draw a two-dimensional space filling curve. Turn in your code and a sample of your output.

2 Methods of Proof (5 points)

Prove that the square of an even number is an even number using

- (a) a direct proof.
- (b) an indirect proof.
- (c) a proof by contradiction.

3 Methods of Proof (5 points)

Prove that if n is an integer and $3n + 2$ is even, then n is even using

- (a) an indirect proof.
- (b) a proof by contradiction.

4 Methods of Proof (5 points)

Use a proof by cases to show that $\lfloor n/2 \rfloor \lfloor n/2 \rfloor = \lfloor n^2/4 \rfloor$ for all integers n .

5 Methods of Proof (5 points)

Prove or disprove that given a positive integer n , there are n consecutive odd positive integers that are primes.

6 Mathematical Induction (5 points)

Find a formula for

$$\frac{1}{1.2} + \frac{1}{2.3} + \dots + \frac{1}{n(n+1)}$$

by examining the values of this expression for small values of n . Use mathematical induction to prove your result.

7 Mathematical Induction (5 points)

Prove that $1.1! + 2.2! + \dots + n.n! = (n+1)! - 1$ whenever n is a positive integer.

8 Mathematical Induction (5 points)

Use mathematical induction to prove that

$$1.2.3 + 2.3.4 + \dots + n(n+1)(n+2) = n(n+1)(n+2)(n+3)/4$$

9 Mathematical Induction (5 points)

Prove that

$$1 + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{n^2} < 2 - \frac{1}{n}$$

whenever n is a positive integer greater than 1.

10 Mathematical Induction (10 points)

Use mathematical induction to prove that a set with n elements has $n(n-1)(n-2)/6$ subsets containing exactly three elements whenever n is an integer greater than or equal to 3.

11 Mathematical Induction (10 points)

Prove that if A_1, A_2, \dots, A_n and B_1, B_2, \dots, B_n are sets such that $A_k \subseteq B_k$ for $k = 1, 2, \dots, n$, then

(a) $\bigcup_{k=1}^n A_k \subseteq \bigcup_{k=1}^n B_k$

(b) $\bigcap_{k=1}^n A_k \subseteq \bigcap_{k=1}^n B_k$

12 Mathematical Induction (5 points)

Use the formula for the sum of the terms of a geometric progression to evaluate the following sums.

- (a) $4 + 4.3 + 4.3^2 + \dots + 4.3^8$
- (b) $3 + 3.2^2 + 3.2^4 + \dots + 3.2^{10}$
- (c) $1 - 2 + 2^2 - 2^3 + \dots + (-1)^n 2^n$

13 Mathematical Induction (10 points)

Find the flaw with the following “proof” that $a^n = 1$ for all nonnegative integers n , whenever a is a nonzero real number.

BASIC STEP: $a^0 = 1$ is true by the definition of a^0 .

INDUCTIVE STEP: Assume that $a^k = 1$ for all nonnegative integers k with $k \leq n$. Then note that

$$a^{n+1} = \frac{a^n \cdot a^n}{a^{n-1}} = \frac{1 \cdot 1}{1} = 1$$

14 Mathematical Induction (10 points)

Show that the following form of mathematical induction is a valid method to prove that $P(n)$ is true for all positive integers n .

BASIC STEP: $P(1)$ and $P(2)$ are true.

INDUCTIVE STEP: For each positive integer n , if $P(n)$ and $P(n+1)$ are both true, then $P(n+2)$ is true.

15 Recursive Definitions (5 points)

Find $f(1)$, $f(2)$, $f(3)$, $f(4)$ and $f(5)$ if $f(n)$ is defined recursively by $f(0) = 3$ and for $n = 0, 1, 2, \dots$

- (a) $f(n+1) = -2f(n)$
- (b) $f(n+1) = 3f(n) + 7$
- (c) $f(n+1) = f(n)^2 - 2f(n) - 2$
- (d) $f(n+1) = 3^{f(n)/3}$