Final Exam
Math 263
Monday, December 16, 2002

Name___________________________

Do all of your work on the blank paper provided. At the end of the exam, hand in your answers with this cover sheet. Include your name on all pages of your exam.

§1 Calculation

1. Consider the statement "Not everybody knows everything."
   a. Write this statement symbolically using quantifiers.
   b. Find the negation of this statement (symbolically)
   c. Write the negation using natural language.

2. Use symbols to write the logical form of the following argument. If it is valid, give the rule that guarantees its validity; if it is invalid specify the fallacy.
   All honest people pay their taxes.
   John is not honest.
   ∴ John does not pay his taxes.

3. Evaluate
   a. $\prod_{i=1}^{4} \frac{i}{2}$
   b. $\sum_{k=1}^{n} \left(\frac{k}{k+1} - \frac{k+1}{k+2} \right)$
   c. $\frac{n!}{(n-3)!3!}$.

4. Let $f : \mathbf{R} \to \mathbf{R}$ be defined by $f(x) = 2x - 9$. Prove that f is bijective, and find f^{-1}.

§2 Comprehension

5. Consider the implication $p \to q$. What is its inverse? What is its converse? What is its contrapositive? Which are equivalent to the original implication?

6. Consider the argument
   You must pay a toll of $2.50 to cross the Bay Bridge
   John paid the $2.50 toll.
   ∴ John crossed the Bay Bridge.
   a. What are the hypotheses?
   b. What is the conclusion?
   c. Is the argument valid or invalid?
   d. Assume the hypotheses are true. Is the conclusion true?

7. Define precisely the following terms.
    a. Statement
    b. Predicate
    c. Modus ponens
    d. Universal Modus tollens
    e. Disjunctive Addition

8. What is the principle of mathematical induction? What is the principle of strong mathematical induction? What is the well-ordering principle?

9. What is the definition of a function? What is the domain, the co-domain, and the range?

§3 Application

10. Prove or disprove: The product of odd integers is odd.

11. Prove or disprove: The square of any odd integer has the form $8m+1$ for some integer m.

12. Prove or disprove: There are an infinite number of primes.

13. Prove $1+r+r^2+r^3+\cdots+r^n=\dfrac{r^{n+1}-1}{r-1}$ for all $r\neq 1$ and for all positive integers n.

14. Prove that every integer greater than 1 is divisible by a prime number.

15. Prove or disprove: For all sets A,B,C, we have $A\times(B\cup C)=(A\times B)\cup(A\times C)$

16. If $f:X\rightarrow Y$ and $g:Y\rightarrow Z$ are functions, and if $(g\circ f):X\rightarrow Z$ is surjective, must f be surjective? Prove or give a counterexample. Must g be surjective? Prove or give a counterexample.

17. Let A and B be sets, and let $f:A\rightarrow B$ be a function. Let $\phi:\wp(A)\rightarrow\wp(B)$ be the function defined by $\phi(U)=f(U)$ for any set $U\subset A$. Show that ϕ is bijective if and only if f is bijective.

18. A frugal student has a part-time job earning s dollars per month. Each month our student deposits that entire amount into a bank account that earns 3% interest per year, compounded monthly.
    a. Write a recursion relation that describes the amount of money b_n present in the account n months after the first deposit.
    b. Solve the resulting recursion relation, and obtain an explicit formula for the amount of money b_n present in the account n months after the first deposit.
    c. If $s=500$, how much will our frugal student have after four years?