Binomial Distribution

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FREC 408

Binomial Random Variables

- In many cases the responses to an experiment are dichotomous
  - Yes/No
  - Alive/Dead
  - Support/Don't Support

Binomial Random Variables

- When our focus is conducting an experiment **n times independently** and observing the number **x of times** that **one** of the **two outcomes occurs**
- This **x** is a Binomial Random Variable
- We can exploit this by using known formulas for a probability distribution

Examples of Binomial Random Variables

- 1,000 people are polled in a telephone survey and asked if they support George W. Bush
  - The responses are Yes (1) or No (0)
  - The proportion saying yes is designated as
    - p
    - (1-p) is the proportion saying No

Binomial Random Variable

- Yes it is a binomial random variable
- Conduct an experiment 1,000 times and observe the number x of times that Yes occurs

Characteristics of a Binomial Random Variable

- The experiment consists of *n* identical trials
- There are only two outcomes on each trial. Outcomes can be denoted as
  - **S for Success**
  - **F for Failure**

Characteristics of a Binomial Random Variable (cont.)

- The probability of S (success) remains the same from trial to trail
  - **Denoted as p the proportion**
- The probability of F (failure)
  - **Denoted as q $q=(1-p)$**
- The trials are independent of each other
- **The binomial random variable x is the number of Successes in *n* trials**
- **Also refer to *Conditions Required for a Binomial Experiment* on P246 of book.**

Example 1: Marketing example

- Marketing survey of 100 randomly chosen consumers
- Record their preferences for a new and an old diet soda – ask them to choose their preference
- Let x be number of 100 who choose the new brand
- **This is a binomial random variable**

Conduct an experiment 100 times and observe the number x of times that Yes occurs

Fitness Example

- Heart Association says only **10%** of adults over 30 can pass the fitness test
- Suppose **4 people** over 30 are selected at random
- Let x be the number who pass the minimum requirements
- Find the probability distribution for x

Conduct an experiment 4 times and observe the number x of times that pass occurs

How to solve the fitness problem – the way we used with discrete random variables

1. List the events
2. List the sample points that refer to that event
3. Calculate the probabilities
   - **$p = .1$ and $q = (1.0 - .1) = .9$**

Event x	Sample Points	Probability
All Fail	FFFF	(.9)(.9)(.9)(.9) = .6561

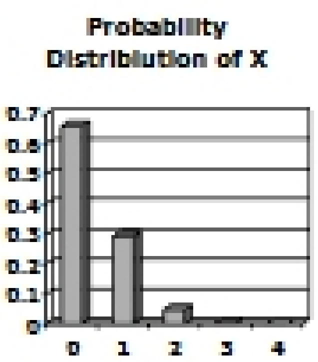
I multiply through on the probabilities because Each trial is independent of the others

Solve for Each Event

Event x	Notation	Probability
0 *All Fail*	FFFF	(.9)(.9)(.9)(.9) **= .6561**
1 *1 Pass 3 Fail*	SFFF FSFF FFSF FFFS	$4[(.1)(.9)^3]$ **=.2916**
2 *2 Pass 2 Fail*	SSFF SFSF SFFS FSSF FSFS FFSS	$6[(.1)^2(.9)^2]$ **=.0486**
3 *3 Pass 1 Fail*	SSSF SFSS SSFS FSSS	$4[(.1)^3(.9)]$ **=.0036**
4 *4 Pass*	SSSS	(.1)(.1)(.1)(.1) **= .0001**

Fitness Example

- When x = 0 All Fail P = .6561
- When x = 1 One Pass P = .2916
- When x =2 Two pass P =.0486
- When x=3 Three pass P = .0036
- When x=4 Four pass P = .0001



Fitness Example

- Find the probability that none of the adults pass the test
  - P(x=0) = .6561
- Find the probability that 3 of 4 adults pass the test
  - P(x=3) = .0036

When we have many trials the formulas get complicated

- We can also use the binomial probability distribution formula
- Using factorial notation
  - n! = n(n-1)(n-2)...(n-(n-1))
  - 5! = 5x4x3x2x1 = 120
  - 0! = 1, 1!=1, 2!=2x1=2, ...
- The formula for any x in n trials is:

$$p(x) = \frac{n!}{x!(n-x)!}(p)^x(q)^{n-x}$$

Binomial Probability Distribution Formula (P248)

$$p(x) = \frac{n!}{x!(n-x)!}(p)^x(q)^{n-x}$$

aka

$$p(x) = \binom{n}{x} p^x q^{n-x}$$

Note: It uses the Combinatorial Rule as the first part of the formula

Most calculators will do all or part of this – become familiar before trying it out

What defines a binomial probability distribution?

- p = Probability of a success on a single trial
- q= (1-p) probability of failure
- n= number of trials
- x = number of successes in n trials

$$p(x) = \frac{n!}{x!(n-x)!}(p)^x(q)^{n-x}$$

$$p(x) = \frac{n!}{x!(n-x)!}(p)^x(q)^{n-x}$$

For x=3 in the fitness example

$$p(3) = \frac{4!}{3!(4-3)!}(.1)^3(.9)^{4-3}$$

$$= \frac{4 \cdot 3 \cdot 2 \cdot 1}{(3 \cdot 2 \cdot 1)(1)}(.001)(.9)$$

$$= \frac{24}{6}(.0009) = .0036$$

This matches the number we generated the other way

Fitness Example Table

Event x	Notation	Probability
0 *All Fail*	FFFF	.6561
1 *1 Pass 3 Fail*	SFFF FSFF FFSF FFFS	.2916
2 *2 Pass 2 Fail*	SSFF SFSF SFFS FSSF FSFS FFSS	.0486
3 *3 Pass 1 Fail*	SSSF SFSS SSFS FSSS	**.0036**
4 *4 Pass*	SSSS	.0001