

Introduction to Dynamical Systems

- Autonomous Planar Systems
- Vector form of a Dynamical System
- Trajectories
- Trajectories Don't Cross
- Equilibria
- Population Biology
 - Rabbit-Fox System
 - Trout System
 - Trout System Phase Portrait

Autonomous Planar Systems

A set of two scalar differential equations of the form

$$(1) \quad \begin{aligned} x'(t) &= F(x(t), y(t)), \\ y'(t) &= G(x(t), y(t)). \end{aligned}$$

is called a **planar autonomous system**.

The term **autonomous** means **self-governing**, justified by the absence of the time variable t in the functions $F(x, y)$, $G(x, y)$.

Vector form of a Dynamical System

To obtain the vector form, let

$$\vec{u}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}, \quad \vec{f}(x, y) = \begin{pmatrix} F(x, y) \\ G(x, y) \end{pmatrix}.$$

Then write system

$$x'(t) = F(x(t), y(t)), \quad y'(t) = G(x(t), y(t))$$

as the first order vector-matrix system

$$(2) \quad \vec{u}'(t) = \vec{f}(\vec{u}(t)).$$

It is assumed that F , G are continuously differentiable in some region D in the xy -plane.