Introduction to Dynamical Systems

- Autonomous Planar Systems
- Vector form of a Dynamical System
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- Trajectories Don't Cross
- Equilibria
- Population Biology
  - Rabbit-Fox System
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Autonomous Planar Systems

A set of two scalar differential equations of the form

$$\begin{aligned} x'(t) &= F(x(t), y(t)), \\ y'(t) &= G(x(t), y(t)). \end{aligned} \tag{1}$$

is called a **planar autonomous system**.

The term **autonomous** means **self-governing**, justified by the absence of the time variable t in the functions $F(x, y), G(x, y)$.

Vector form of a Dynamical System

To obtain the vector form, let

$$\vec{\mathrm{u}}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}, \quad \vec{\mathrm{f}}(x, y) = \begin{pmatrix} F(x, y) \\ G(x, y) \end{pmatrix}.$$

Then write system

$$x'(t) = F(x(t), y(t)), \quad y'(t) = G(x(t), y(t))$$

as the first order vector-matrix system

$$\vec{\mathrm{u}}'(t) = \vec{\mathrm{f}}(\vec{\mathrm{u}}(t)). \tag{2}$$

It is assumed that F, G are continuously differentiable in some region D in the xy-plane.